Some features of α disc and advection-dominated accretion disc. Self-similar solutions and their comparison -I

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Abstract

A brief review of the features of Standard Shakura - Sunyaev Disc (SSD) and Advection - dominated Accretion Disc (ADAD) is discussed. In this paper, it is presented the physical bases, which we use to obtain the parameters, describing two models. The built theoretical systems are transformed in a suitably for operation view.

1. Introduction

The new, more functional theory about disc accretion - the advection theory [10], has appeared in the last years.

It has arisen because of that the standard theory gives common view on accretion flows, but couldn't explain any observant phenomena as: very high effective temperature (in standard theory disc is unstable - transforms to tore); non - thermal spectrum with power dependence of luminosity L from accretion rate $\dot{M}$ ($\sim M^2$ in two - temperature model); jets and s.o.

Other priority is that the advection - dominated flows may occur in both cases of optical depth - very large or very small it's value [10], which extend the volume of studying objects: active galactic nuclei, elliptical galactics, X-ray binaries and cataclysmic variables.

The conditions of transition between standard Shakura - Sunyaev disc and Advection - dominated disc are discussed by Abramowicz and Igumenchev [1]. They used a simple two - dimensional hydrodynamical model, assuming an instant destruction of SSD by some unknown physical process at radius r_{trns}. The result of their investigation shows that flux of matter from the destroyed SSD expands and forms thick disc (ADAF). The energy, which is necessary for expansion, is supplied locally by viscous heating. So expanded matter flows in all direction from source of matter and forms a geometrically thick disc.

Yamasaki [13] investigates the stability of two - dimensional ADAD against local thermal perturbations - for optically thin discs. In result he obtains that weakly unstable modes exist due to radiation effects, but the mode is stable when the thermal conduction is efficient. Because of turbulent heat diffusion, in two - temperature ADAF thermal perturbations damp.

Wu [12] proved, that in the case of very small advection, thermal instability exist when the disc is geometrically thin. If consider thermal diffusion, however it disappears. More than if the disc is advective - dominated thermal instability doesn't exist. There are enough dates that advection and thermal diffusion have significant effect on the stability of hot optically thin disc. The detail stability analysis of Wu shows that only two stable thermal equilibrium of accretion disc exist. One of them is optically thin advection - dominated and the other is optically thick gas - dominated.

The family of self - similar solutions [10], where the temperature of accreting gas is almost virial and flow is quasi - spherical, define some of properties of the ADAF, as:

- The angular velocity of the flow Ω is less then Keplerian angular velocity Ω_k.
- ADAF is convective instable, because convection transfers energy from small to large radius.
- Bernoulli parameter b (scale changed) is positive in self - similar ADAF for wide range of parameters, e.g. gas may spontaneously expand to infinity.

Nakamura [9] elaborates global steady models of two - temperatures, advection - dominated accretion flows around black holes, as he pays attention to transonic region near black hole.

Chen and Abramowicz [4] present optically thin ADAD, described by full system of differential equations. They obtain global transonic solutions. As a result from this follows that far from sonic point, self - similar solutions is a good approximation to global structure of the flow. That is true if accretion rate is close to maximum value, above, which the solutions for optically thin disc don't exist. The simple self-similar solutions nowhere approach to complete solution [11].

In recent work we consider optically thick advection - dominated flows. The mainly aim of the paper is to show that the optically thin disc remains geometrically thin because of the advection [3]. It is known that when α increases the sonic point removes outward [4]. That is why such advective flow is supersonic, when viscosity parameter α is large for optically thick disc.

This letter is built as follows: It consists of two parts.

Part I: In § 2 we present the main physically characteristics of two flows. In § 3 is present the vertical structure. Comments of part I.

Part II: Paragraph 4 describes the equations of evolution of both discs. In § 5 we have obtained the self-similar solutions. Discussion of part II.

2. **Basic equations.**

We can use cylindrical system of coordinates because of the form of accretion flows. The potential create in φ acceleration in form:

$$(2.1) \quad \frac{V_\varphi}{r} = \frac{d\Phi}{dr}$$

where V_φ is the linear velocity in φ.

(2.2) $V_\varphi = \omega r$

We use Newtonian gravitational potential for Standard disc:

$$(2.3) \quad \Phi = \frac{GM}{r}$$

and pseudo- Newtonian $\Phi = \dfrac{GM}{r - r_g}$ (2.4) for Advective disc,

where

(2.5) $r_g = \dfrac{2GM}{c^2}$ is gravitational radius of the black hole .

G - gravitational constant, M - mass of the central object, c - the light speed.

The angular velocities for both discs are:

$$(2.6) \quad \omega_k = \sqrt{\frac{GM}{r^3}}$$

$$(2.7) \quad \omega = \sqrt{\frac{GM}{r\left(r - r_g\right)^2}}$$

Geometrically slim discs are described with another one parameter - surface density of the disc:

(2.8) $$\Sigma = \int_{-H}^{H} \rho dz \cong 2H\rho$$

Now we can form the basic equations of non-stationary accretion:

<u>The mass conserve law:</u>

(2.9) $$r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}\left(r\Sigma V_r\right) = 0$$

There isn't any different in equation for two discs, but V_r is much larger in advective one.

<u>The conservation law of momentum:</u>

(2.10) $$r\frac{\partial}{\partial t}\left(\Sigma r^2 \omega\right) + \frac{\partial}{\partial r}\left(r\Sigma V_r r^2 \omega\right) = \frac{1}{2\pi}\frac{\partial \theta}{\partial r}$$

θ is the momentum by viscosity forces:

(2.11) $\theta = 2\pi\, W_{r\varphi}\, r^2$

$W_{r\varphi}$ - vertically integrated viscosity per unit length by circum.

(2.12) $$W_{r\varphi} = \int_{-H}^{H} \omega_{r\varphi} dz = \nu\Sigma r\frac{\partial \omega}{\partial r}$$

ν - kinetic viscosity;

(2.13) $\nu = \alpha V_s H$

V_s is the sound velocity,

$r\frac{\partial \omega}{\partial r}$ is a displacement between two layers in differential rotation of the disc.

<u>Thermal balance equation</u>

The discs are optically thick and for that reason the Local thermodynamically equilibrium exist.

(2.14) $Q^+ \sim Q^-$

where Q^+ is the heating produced by viscosity:

(2.15) $$Q^+ = \frac{1}{2} W_{r\varphi}\left(r\frac{\partial \omega}{\partial r}\right)$$ and

(2.16) $Q^{-} = \frac{acT^4}{\tau}$ is radiated cooling

a - radiation constant.

T - effective temperature

τ - optically depth of the disc

(2.17) $d\tau = \rho\chi dz$

χ - opacity coefficient.

However if the rate of accretion is increased and the inflow's time become shorter than the time of photon emission the disc cannot reradiate the generated energy. Part of radiation caught by flow generated energy, which decrease the gradient of entropy and the flow direct to the center. Thereby the advection is appeared and the thermal balance takes the form:

(2.18) $Q_{adv} + Q^{+} = Q^{-}$

where

(2.19) $Q_{adv} = \Sigma V_r T \frac{dS}{dr}$

and $\frac{dS}{dr}$ radial gradient of the entropy.

Equations of radial motion:

(2.20) $\Sigma \frac{\partial V_r}{\partial t} + \Sigma V_r \frac{\partial V_r}{\partial r} - \Sigma\omega_k^2 r = W_{r\varphi} + G$

(2.21) $\Sigma \frac{\partial V_r}{\partial t} + \Sigma V_r \frac{\partial V_r}{\partial r} - \Sigma\left(\omega^2 - \omega_k^2\right) r = -2\frac{\partial Hp}{\partial r} + W_{r\varphi} + G$

(2.22) $P = \int_{-H}^{H} Pdz$

(ω^2 - $\omega^2_к$) - the equilibrium in the disc changed in result of advection . The inertial spring is needed to keep the structure stable.

Using the similar system (2.9, 2.10, 2.18, 2.21) Narayan and Yi [10] obtained that in advective disc:

$$(2.23)\quad \begin{aligned} V_r &= -c_1^{'}\omega_k r \\ \omega &= \omega_k c_2 \\ V_s &= c_3\omega_k^2 r^2 \end{aligned}$$

where $c'_1\, c_2\, c_3$ are dimensionless constants.

3.Vertical structure [8].

Equation of hydrostatical equilibrium:

(3.1) $\frac{1}{\rho}\frac{\partial P}{\partial z} = -\omega_k^2 z$

(3.2) $\frac{1}{\rho}\frac{\partial P}{\partial z} = -\omega^2 z$

Equation of continuity:

(3.3) $\frac{\partial \Sigma}{\partial z} = \rho$

$\Sigma = 2H\rho$

Equation of radiation transfer:

(3.4) $\frac{c}{3\chi\rho}\frac{\partial\left(aT^4\right)}{\partial z} = -Q^+$

(3.5) $-\Sigma V_r T\frac{\partial S}{\partial z} + \frac{c}{3\chi\rho}\frac{\partial\left(aT^4\right)}{\partial z} = -Q^+$

and we take into account (2.23):

(3.6) $\frac{c_1'}{\sqrt{c_3}} V_s \Sigma T\frac{dS}{dz} + \frac{c}{3\chi\rho}\frac{\partial\left(aT^4\right)}{\partial z} = -Q^+$

where

(3.7) $S = c_p \ln T - R\ln P$

is the entropy of ideal gas, R is the gas constant.

The vertical gradient of radiating fluctuation is equal to energy released in disc:

(3.8) $\frac{\partial Q}{\partial z} = \varepsilon$

Equation of ideal gas:

(3.9) $P = \rho \frac{RT}{\mu}$

$\frac{P}{\rho} = V_s^2$

χ is opacity coefficient:

(3.10) $\chi = \frac{\chi_0 \rho^a}{T^b}$

χ_0 Thompson opacity coefficient:

a, b are constants.

The transformation of obtained differential system we will make by appropriate group, corresponding to the slim disc model approximation:

(3.11) ΔP ~ -P; ΔQ ~ Q; ΔT ~ T; ΔZ ~ H

This allows us to receive the solution in power dependence of independent variables or their dimensionless combination – that is the self-similar solution [2].

However to obtain the full algebraic system we must include specifically moments in discs:

(3.12) $h_* = \omega r^2$

as well as the average momentum of viscosity power between the disc's adjacent payers.

We have obtained the system of equations for both discs as follows:

Standard disc	Equations refer to both discs	Advection-dominated disc
$h = \sqrt{GMr}$ (3.13)		$h_* = \frac{\sqrt{GMr^3}}{r - r_g}$ (3.14)
$\omega_k = \sqrt{\frac{GM}{r^3}}$ (3.15)		$\omega = \sqrt{\frac{GM}{r\left(r - r_g\right)}}$ (3.16)
$V_s = \omega_k H$ (3.17)	$P = \frac{\Sigma}{2H} V_s^2$ (3.18)	$V_s = \omega H$ (3.19)
$W_{r\varphi} = k \Sigma T$ (3.20)	$F = W_{r\varphi} r^2$ (3.21)	$W_{r\varphi} = k^{'} \Sigma T$ (3.22)

$k = -\frac{3}{2}\alpha\frac{R}{\mu}$		$k^{'} = -\left(\frac{1}{2} + c_2\right)\alpha\frac{R}{\mu}$
$Q^{+} = -\frac{3}{4}W_{r\varphi}\omega_k$ (3.23)		$Q^{+} = -\frac{1}{2}\left(\frac{1}{2} + c_2\right)W_{r\varphi}\omega$ (3.24)
$\varepsilon = aT^4 = -\frac{3\chi\Sigma}{2c}Q^{+}$ (3.25)		$\frac{c_1^{'}}{\sqrt{c_3}}\frac{V_s\Sigma T}{H}\left(c_p - R\right) + \frac{2acT^4}{3\chi\Sigma} = -Q^{+}$ (3.26)
$\chi = k_1\Sigma^{a_1}T^{b_1}H^{c_1}$ (3.27) $k_1 = \frac{\chi_0}{2^{a_1}}$		$\chi = k_{1a}\Sigma^{a_{10}}\ T^{b_{10}}H^{c_{10}}$ (3.28) $k_{1a} = \frac{\chi_0}{2^{a_{10}}}$

This algebraically system can be solved as in (2.12) we take $\bar{\nu}$ in advective case and we use (2.7) и (3.12). We will obtain the parameters of both discs in dependent only of their Σ and ω_k and the average viscosity moments from Σ and h, too [6].

The different solutions for both discs are obtained:

Standard α disc	Advection-dominated disc
$T = T_0\Sigma^{2N_1}\omega_k^{2N_2}$ (3.29) $T_0 = \left\{\frac{-27\alpha\chi_0}{2^{a_1+4}ac}\left(\frac{R}{\mu}\right)^{\frac{c_1+2}{2}}\right\}^{\frac{2}{6-2b_1-c_1}}$ $N_1 = \frac{a_1+2}{6-2b_1-c_1}$ $N_2 = \frac{1-c_1}{6-2b_1-c_1}$	$T = T_0^{a}\omega_k^{2}r^2$ (3.34) $T_0^{a} = \frac{W_{r\varphi 0}^{a}}{k^{'}}$
$V_s = V_{s0}\Sigma^{N_1}\omega_k^{N_2}$ (3.30) $V_{s0} = \left(\frac{RT_0}{\mu}\right)^{\frac{1}{2}}$	$V_s = V_{s0}^{a}\omega_k r$ (3.35) $V_{s0}^{a} = \left(\frac{RT_0^{a}}{\mu}\right)^{\frac{1}{2}}$

$W_{r\varphi} = W_{r\varphi 0}\Sigma^{2N_1+1}\omega_k^{2N_2}$ (3.31) $W_{r\varphi 0} = \dfrac{-3\alpha RT_0}{2\mu}$	$W_{r\varphi} = W^a_{r\varphi 0}\Sigma\omega_k{}^2 r^2$ (3.36) $W^a_{r\varphi 0} = \alpha c_3$
$P = P_0\Sigma^{N_1+1}\omega_k{}^{N_2+1}$ (3.32) $P_0 = \left(\dfrac{RT_0}{4\mu}\right)^{\frac{1}{2}}$	$P = P_0^a\Sigma\omega_k{}^2 r$ (3.37) $P_0^a = \left(\dfrac{V^a_{s0}c_2}{2}\right)$
$F = F_0\Sigma^A h^B$ (3.33) $F_0 = W_{r\varphi 0}(GM)^{\frac{-8+4b_1-2c_1}{6-2b_1-c_1}}$ $A = \dfrac{10+2a_1-2b_1-c_1}{6-2b_1-c_1}$ $B = \dfrac{18-8b_1+2c_1}{6-2b_1-c_1}$	$F = \left(2\dfrac{h_*}{h} - \dfrac{1}{2}\dfrac{\partial h_*}{\partial h}\right)h\Sigma\bar{\nu}$ (3.38)

4. Discussion

In the paper are shown the main theoretical principles when there is a development of the accretion in a standart and advection regimes. It's formed the horizontal and vertical structures of the accretion discs in two regimes, when the geometrically thin disc approximation is conserved.

We have emphasized on the processes, which determine the behaviour of the disc plasma in two considered cases.

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