Some features of α disc and advective-dominated accretion disc. Self-similar solutions and their comparison -II

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Abstract

Using the models from part I, we have derived the basic parameters, describing the discs. We have obtained the self-similar solutions of the evolution for both types - ADAD and α discs. The results are expressed also quantitative to be demonstrated our conclusion.

1. Introduction

As a consequence of part I, where we have mentioned about the priority of advection theory and the properties of advection - dominated flow with the comparison of the Standard accretion theory, here we present the really result of our calculations.

In many problems the simple self-similar solutions don't correspond to complete solution [12]. They are intervening asymptotic and in a number of cases they give a sufficient idea about the researching physical phenomena with correctly boundary conditions.

As a result of necessary transformation in our letter and we make use of work [7], with adequate variables, we will obtain a self-similar solutions, too.

2. Equations, describing the evolution of α disc and ADAD.

The last two systems from part I [8] (eq. 3.29 ÷ 3.33; 3.34 ÷ 3.38) enable us to obtain all parameters of the disc and we are searching only Σ in obvious form. And for that we will use the conserve laws and using (eq. 2.10, see [8]) we obtain:

$$\text{(II.1)}\quad \Sigma V_r r = \frac{\overset{\circ}{M}}{2\pi} = -\left(\frac{\partial h_*}{\partial h}\right)^{-1} \frac{\partial F}{\partial h}$$

which gives respectively:

$$\text{(II.2)}\quad \dot{M} = -2\pi \frac{\partial F}{\partial h}$$

$$\text{(II.3)}\quad \dot{M} = -\frac{2\pi}{c_2} \frac{\partial F}{\partial h}$$

From (eq. 2.9, see [8]) and (II.1) come after:

$$(\text{II.4}) \quad \frac{\partial \Sigma}{\partial t} = \frac{1}{2}\frac{(GM)^2}{L^3}\frac{\partial}{\partial h}\left\{ \left(\frac{\partial h_*}{\partial h} \right)^{-1} \frac{\partial F}{\partial h} \right\}$$

as we apply (eq. (3.33) and eq. (3.38) see [8]) and the relation $\frac{\bar{v}}{h} = \left(\frac{2\alpha c_3}{3c_2} \right)$ we obtain the follows diffusion equations:

$$(\text{II.5}) \quad \frac{\partial F}{\partial t} = \Pi \frac{F^m}{h^n}\frac{\partial^2 F}{\partial h^2}$$

$$(\text{II.6}) \quad \frac{\partial F}{\partial t} = \frac{\Pi_a}{h}\frac{\partial^2 F}{\partial h^2}$$

where:

$$\Pi = \frac{AF^{\frac{1}{A}}}{2}(GM)^2 \qquad \Pi_a = \frac{\alpha}{2}\frac{c_3}{c_2}(GM)^2$$

$$m = \frac{4 + 2a_1}{10 + 2a_1 - 2b_1 - c_1} \qquad n = \frac{12 + 6a_1 + 2b_1 - 5c_1}{10 + 2a_1 - 2b_1 - c_1}$$

From (II.4) we get:

$$(\text{II.7}) \quad \Sigma = \frac{(GM)^2 F^{1-m}}{2(1-m)\Pi h^{3-n}}$$

$$(\text{II.8}) \quad \Sigma = \frac{(GM)^2}{2\Pi_a}\frac{F}{h^2}$$

3. Self – similar solutions.

First we define the role of self-similar solutions and we will show a model of their application. Such model is examination of the temperature diffusion equation for stationary conductive medium, presented in [7]:

$$\frac{\partial T}{\partial t} = D\nabla T$$

D - constant of diffusion.

It should to determine the temperature in succssive moments, if the initial distribution is: $T = Kr^x$, r - the to the centre of the coordinate system.

If we define the scale of the temperature U , the distance Λ and the time η, then we can determine dimensions D and K :

$[D] = \eta^{-1}\Lambda^2$ and $[K] = \Lambda^{-x}U$

D independent of U

Sometimes, after the beginning of the process, the typical length scale depending on the time may be defined as:

$$\Lambda_c(t) = (Dt)^{1/2}$$

The time-depending temperature scale may be defined in a similar way:

$$T_c(t) = K\Lambda_c(t)^x$$

The solution should yield T as a function of t and r. In non-dimensional form:

$$\frac{T}{T_c} = \frac{T}{K\Lambda^x}$$

This form should be a function of $\frac{r}{\Lambda_l(t)}$ and $\frac{t}{t}$.

So we obtain the solution in the form :

$$T = K\Lambda_c{}^x T_* \left(\frac{r}{\Lambda_c(t)} \right)$$

T_* - dimensionless function composed of its dimensionless arguments.

The obtained result is a self-similar solution, since time dependent scales are used. The temperature scale ia always the function of scale featuring the length. This is the self-similarity of the problem which denote that variable scales of Λ_c and T_c may be selected. Because of this, there is a possibility to represent the scale of characteristics by a single variable function.

Therefore, the present of several dimensions of the independent constants, including the boundary conditions of the problem defines the nccesserity of the self-similar solution.

Let us examine the problem where the self-similar solution is of first order. The time behavior of a thin disc is defined by (II.5) at the assumption that for the initial moment $t = 0$ the distribution is:

$$F = Kh^y$$

The dimensions of all values in (II.5) and initial conditions are:

$$[h] = \Lambda^2\eta^{-1}; [t] = \eta\,; \ [F] = M\Lambda^2\eta^{-2};$$
$$[\Pi] = M^{-m}\Lambda^{-2(n-m+2)}\eta^{2n-m-3};$$
$$[K] = M\Lambda^{2(1-y)}\eta^{y-2}.$$

Now we have to determine the typical scale of the total angular momentum $h_c(t)$ and typical scale of friction $F_c(t)$ for each moment $t > 0$.

The first value is yielded by the dimensional analysis of (II.5):

$$h_c(t) = \left(\Pi F_c(t)^m t\right) \frac{1}{n+2}$$

For $F_c(t)$ we use the initial distribution:

$$F_c(t) = Kh_c(t)^y$$

Subtituting the last equation in the upper one, we obtain for h_c:

$$h_c(t) = \left(\Pi K^m t\right) \frac{1}{n+2-xm}.$$

The solution of the problem yields F as a function of h and t and may be expressed in dimensionless form:

$$\frac{F}{F_c} = \frac{F}{Kh_c(t)^y} = F_*\left(\frac{h}{h_c}.\frac{t}{t}\right) = F_*\left(\frac{h}{h_c}\right)$$

Then the function F will take the form:

$$F(h.t) = Kh_c^{\ y}(t)F_*\left(\frac{h}{h_c}\right).$$

Using the present above [7], we divide the variables in (II.5) and (II.6):

$$F(th) = F(t)f(\xi),\ \xi = \frac{h}{h_0} = \sqrt{\frac{r}{r_{out}}};\quad h_0 = \sqrt{GMr_{out}}$$

r_{out} is the edge of the disc.

Then:

$$\text{(III.1)}\quad F(t) = \left[\frac{h_0^{\ n+2}}{-\lambda m\Pi(t+t_0)}\right]^{\frac{1}{m}}$$

$$\text{(III.2)}\quad F(t) = F_0 e^{\beta t};\ \beta = \frac{\lambda_a \Pi_a}{h_0^{\ 3}}$$

$$\text{(III.3)}\quad \frac{\partial^2 f}{\partial \xi^2} = \lambda \xi^n f^{1-m}$$

$$\text{(III.4)}\quad \frac{\partial^2 f}{\partial \xi^2} = \lambda_a \xi f$$

Also, we can search the function in polynomial form:

$$\text{(III.5)}\quad f(\xi) = a_0\xi + a^{'}\xi^{l} + a_2\xi^{l_1}$$

$$l = 3+n-m,\quad a^{'} = \frac{\lambda a_0^{\ 1-m}}{l(l-1)},\quad a_2 = \frac{\lambda^2 a_0^{\ 1-2m}(1-m)}{2l(2l-1)(l-1)^2}$$

$$a_0 = \frac{2l+1}{2(l-1)},\quad \lambda = l(2l-1)a_0^{\ m-1} - 2l(l-1)a_0^{\ m}$$

and the boundary conditions are:

$$f(1) = 1 \qquad f(0) = 0$$
$$f^{'}(1) = 0$$

finally we replace (III.1), (III.2), (III.5) in (II.7) and (II.8)

Then we replace the result in ((eq. 3.29 ÷ 3.32,) and (eq. 3.34 ÷ 3.37), see [8]). As a result the parameters of two discs are obtained in obvious form of time and dimensionless coordinate ξ.

Standard accretion disc:

$$\dot{M} = \frac{\dot{M}_k f^{'}}{\Psi^{\frac{1}{m}}}; \qquad \dot{M}_k = \frac{-2\pi}{h_0}\left(\frac{h_0^{\ n+2}}{-\lambda m \Pi t_\phi}\right)^{\frac{1}{m}}; \quad \Psi = \frac{t + t_0}{t_\phi}$$

$$\frac{\Sigma}{\Sigma_k} = \Psi^{\frac{m-1}{m}} f^{1-m} \xi^{n-3}; \qquad \Sigma_k = \frac{1}{2}\frac{(GM)^2}{h_0^{\ 3-n}}\frac{1}{\Pi(1-m)}\left(\frac{h^{n-2}}{-\lambda m \Pi t_\phi}\right)^{\frac{1-m}{m}}$$

$$\frac{T}{T_k} = \Psi^{2N_1\frac{m-1}{m}} f^{2N_1(1-m)} \xi^{2N_1(n-3)-6N_2}; \qquad \begin{aligned} T_k &= T_0 \Sigma_k^{\ 2N_1} \omega_{k0}^{\ 2N_2} \\ \omega_{k0} &= \omega_k\left(r_{out}\right) \end{aligned}$$

$$\frac{V_s}{V_{sk}} = \Psi^{N_1\frac{m-1}{m}} f^{N_1(1-m)} \xi^{N_1(n-3)-3N_2}; \qquad V_{sk} = V_{s0} \Sigma_k^{\ N_1} \omega_{k0}^{\ N_2} \tag{III.6}$$

$$\frac{W_{r\varphi}}{W_{r\varphi}^{\ k}} = \Psi^{(2N_1+1)\frac{m-1}{m}} f^{(2N_1+1)(1-m)} \xi^{(2N_1+1)(n-3)-6N_2}; \quad W_{r\varphi}^{\ k} = W_{r\varphi 0} \Sigma_k^{\ 2N_1+1} \omega_{k0}^{\ 2N_2}$$

$$\frac{P}{P_k} = \Psi^{(N_1+1)\frac{m-1}{m}} f^{(N_1+1)(1-m)} \xi^{(N_1+1)(n-3)-3(N_2+1)}; \qquad P_k = P_0 \Sigma_k^{\ N_1+1} \omega_{k0}^{\ (N_2+1)}$$

$$\frac{\tau}{\tau_k} = \Psi^{q_1\frac{m-1}{m}} f^{(1-m)q_1} \xi^{q_1(n-3)-3q_2}; \quad \begin{aligned} \tau_k &= k_1 T_0^{\ b_1} V_{s0}^{\ c_1} \Sigma_k^{\ q_1} \omega_k^{\ q_2} \\ q_1 &= a_1 + 1 + \left(2b_1 + c_1\right) N_1 \\ q_2 &= \left(2b_1 + c_1\right) N_2 - c_1 \end{aligned}$$

$$L = \sigma \dot{M}(0,t) c^2; \quad \frac{L}{L_E} = \frac{\sigma \dot{M}_k \left[M_\circ / day\right]}{L_E} \frac{a_0 c^2}{\Psi^{\frac{1}{m}}}$$

Advection - dominated discs:

$$\dot{M} = \dot{M}_a e^{\beta t} f^{'}; \qquad \dot{M}_a = \frac{-2\pi}{c_2}\frac{F_0}{h_0}$$

$$\frac{\Sigma}{\Sigma_a} = e^{\beta t} f \xi^{-2}; \qquad \Sigma_a = \frac{F_0}{c_3 \alpha h_0^{\ 2}}$$

$$\frac{T}{T_a} = \xi^{-2}; \qquad T_a = T_0^{\ a} \omega_{k0}^{\ 2} r_{out}^{\ 2}$$

$$\frac{V_s}{V_{sa}} = \xi^{-1}; \qquad V_{sa} = V_{s0}^{\ a} \omega_{k0} r_{out}$$

$$\begin{aligned}
&\frac{W_{r\varphi}}{W_{r\varphi a}} = e^{\beta t} f \xi^{-4}; \qquad && W_{r\varphi a} = W_{r\varphi 0}{}^{a}\Sigma_a \omega_{k0}{}^{2} r_{out}{}^{2} \\
&\frac{P}{P_a} = e^{\beta t} f \xi^{-6}; \qquad && P_a = P_0{}^{a}\Sigma_0 \omega_{k0}{}^{2} r_{out} \\
&\frac{\tau}{\tau_a} = e^{\beta t} f \xi^{-2}; \qquad && \tau_a = \chi_0 \Sigma_a \\
&\frac{L}{L_E} = \sigma_a \frac{\dot{M}_a\left[M_\circ / day\right]}{L_E} a_a c^2 e^{\beta t} &&
\end{aligned} \tag{III.7}$$

After using the tables 1 and 2 (Appendix 1), and from (III.6) we obtain the parameters of the disc about two regimes: Thompson's opacity and free - free transition.

To obtain the parameters of advective disc we must define the constants c_2, c_3. Using the equations

$$\omega = c_2 \omega_k; \quad h = \omega_k r^2; \quad h_* = \omega r^2; \quad \frac{\partial h_*}{\partial h} = c_2$$

we can find c_2.

But the value of c_3 we can not define exactly, we can give an appreciation only and taking into account physically and mathematically conditions. To keep the slim disc formation it is necessary $\frac{H}{r}$ doesn't exceed 10^{-2} . But the disc is advective hot and then H/r is in maximum, that is why we consider $\frac{H}{r} = 10^{-2}$.

4. Discussion

It has been made a comparison between standard and advective model of the accretion disc and in the result has obtained the main parameters of both discs in dimensionless quantities. We have used hydrodynamical equations, as we have added the terms describing the advection. The obtained solutions are self-similar.

The results (Appendix 2) lay down the field of action of the two theory and they prove that the new advective theory can be use, while the mainly priority of the standard theory - the slim disc approximation, remains.

The presentation enables us to yield the full approximation solution of parameters of the disc at non-stationary accretion. Although, the self-similar solution doesn't accurately, it shows good quality estimation about physically processes in a given astrophysical disc.

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НЯКОИ ОСОБЕНОСТИ НА α ДИСК И АДВЕКТИВНО - ДОМИНИРАЩ АКРЕЦИОНЕН ДИСК. АВТОМОДЕЛНИ РЕШЕНИЯ И ТЯХНОТО СРАВНЕНИЕ - II

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Резюме

На базата на структурираните модели в част I, са изведени основните параметри, характеризиращи двата диска. Получени са автомоделни решения за еволюцията на двата типа -Адвективно-доминиращ и α диск. Резултатите са представени и количествено за да потвърдят нашите изводи.

APPENDIX 1

Table No.1

Regime	a_1	b_1	c_1
χ_0	0	0	0
χ_{ff}	1	-3,5	-1
α	$M/M_\circ$	r_{out}/R_o	μ
0,3	3	1	0,5

Table No.2

Regime	**m**	**n**	λ	a_0	$a^{'}$	a_2	**l**	l_1	N_1	N_2	q_1	q_2
χ_0	2/5	1,2	3,482	1,376	-0,39	0,02	3,8	6,6	1/3	1/6	1	0
χ_{ff}	0,3	0,8	3,137	1,430	-0,46	0.03	3,5	6,0	3/14	1/7	2/7	-1/7

Table No.3

c_1	c_3	λ_a	a_a	$a^{''}$	a_3	γ	γ_1	$\beta\left[\frac{1}{d}\right]$
1	10^{-4}	-5,33	1,5	-0,66	0.08	4	7	$7{,}46.10^{-3}$

APPENDIX 2

Table No.4

Σ_T/Σ_k	**Σ_{ff}/Σ_k**	**Σ/Σ_{ad}**	ξ	**$\lg\Sigma/\Sigma_k$**	**$\lg\Sigma/\Sigma_{ad}$**	Δ
517,15		150,00	0,01	2,71	2,18	0,8
32,73		14,99	0,1	1,51	1,18	0,08
14,18		7,47	0,2	1,15	0,87	0,07
$\Delta=10^{-2}$	7,73	4,94	0,3	0,89	0,69	0,05
	4,96	3,64	0,4	0,70	0,56	0,04
	3,49	2,93	0,5	0,54	0,47	0,03
	2,59	2,27	0,6	0,41	0,36	0,03
	1,99	1,83	0,7	0,30	0,26	0,03
	1,57	1,48	0,8	0,19	0,17	0,03
	1,25	1,18	0,9	0,10	0,07	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.5

T_t/T_k	**T_{ff}/T_k**	**T/T_{ad}**	ξ	**$\lg T/T_k$**	**$\lg T/T_{ad}$**	Δ
6445,6		10^4	0,01	3,81	4,00	0,8
102,14		10^2	0,1	2,01	2,00	0,08
29,30		25,00	0,2	1,47	1,40	0,07
$\Delta=10^{-2}$	6,74	11,11	0,3	0,83	1,04	0,05
	4,36	6,25	0,4	0,64	0,80	0,04
	3,09	4,00	0,5	0,49	0,60	0,03
	2,33	2,78	0,6	0,37	0,44	0,03
	1,82	2,04	0,7	0,26	0,31	0,03
	1,46	1,56	0,8	0,17	0,19	0,03
	1,20	1,23	0,9	0,08	0,09	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.6

V_s^T/V_{sk}	V_s^{ff}/V_{sk}	V_s/V_{sad}	ξ	lgV_s/V_{sk}	lgV_s/V_{sad}	Δ
80,03		10^2	0,01	1,90	2,00	0,8
10,07		10	0,1	1,00	1,00	0,08
5,40		5,00	0,2	0,73	0,70	0,07
$\Delta=10^{-2}$	2,60	3,33	0,3	0,41	0,52	0,05
	2,09	2,50	0,4	0,32	0,40	0,04
	1,76	2,00	0,5	0,24	0,30	0,03
	1,53	1,67	0,6	0,18	0,22	0,03
	1,35	1,43	0,7	0,13	0,15	0,03
	1,21	1,25	0,8	0,08	0,10	0,03
	1,10	1,11	0,9	0,04	0,04	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.7

$W_{r\varphi}^T/W_{r\varphi}^k$	$W_{r\varphi}^{ff}/W_{r\varphi}^k$	$W_{r\varphi}/W_{r\varphi}^{ad}$	ξ	$lgW_{r\varphi}/W_{r\varphi}^k$	$lgW_{r\varphi}/W_{r\varphi}^{ad}$	Δ
$3,3.10^6$		$1,4999.10^6$	0,01	6,52	6,18	0,8
3301,0		1498,3	0,1	3,52	3,18	0,08
411,51		186,84	0,2	2,61	2,27	0,07
$\Delta=10^{-2}$	52,13	54,90	0,3	1,72	1,74	0,05
	21,62	22,78	0,4	1,33	1,38	0,04
	10,80	11,74	0,5	1,03	1,07	0,03
	6,04	6,30	0,6	0,78	0,80	0,03
	3,63	3,74	0,7	0,56	0,57	0,03
	2,30	2,31	0,8	0,36	0,36	0,03
	1,50	1,46	0,9	0,18	0,16	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.8

P_T/P_k	P_{ff}/P_k	P/P_{ad}	ξ	$\lg P/P_k$	$\lg P/P_{ad}$	Δ
$4{,}09.10^{10}$		$1{,}4999.10^{10}$	0,01	10,61	10,18	0,8
$3{,}25.10^{5}$		$1{,}4993.10^{5}$	0,1	5,51	5,18	0,08
9453,30		4671,00	0,2	3,98	3,67	0,07
$\Delta=10^{-2}$	743,48	6,	0,3	2,87	2,78	0,05
	161,85	142,39	0,4	2,21	2,15	0,04
	49,10	46,96	0,5	1,69	1,67	0,03
	18,31	17,50	0,6	1,26	1,24	0,03
	7,84	7,63	0,7	0,89	0,88	0,03
	3,90	5,61	0,8	0,59	0,56	0,03
	1,88	1,80	0,9	0,27	0,25	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.9

τ_T/τ_k	τ^{ff}/τ_k	τ/τ_{ad}	ξ	$\lg\tau/\tau_k$	$\lg\tau/\tau_{ad}$	Δ
517,15		150,00	0,01	2,71	2,18	0,8
32,73		14,19	0,1	1,51	1,18	0,08
14,18		7,47	0,2	1,15	0,87	0,07
$\Delta=10^{-2}$	1,07	4,94	0,3	0,03	0,69	0,05
	1,07	3,64	0,4	0,03	0,56	0,04
	1,06	2,93	0,5	0,03	0,47	0,03
	1,05	2,27	0,6	0,03	0,36	0,03
	1,04	1,83	0,7	0,02	0,26	0,03
	1,03	1,48	0,8	0,02	0,17	0,03
	1,02	1,18	0,9	0,01	0,07	0,03
	1,00	1,00	1,0	0,01	0,00	0,03

Table No.10

$(\Sigma_k/\Sigma_0)_\tau.10^{-2}$	$(\Sigma_k/\Sigma_0)_{ff}.10^{-4}$	Σ_{ad}/Σ_a	$t_\phi[d]$
1,12	9,30	0,86	20
0,61	3,62	0,80	30
0,40	1,85	0,74	40
0,28	1,10	0,69	50
0,22	0,72	0,64	60
0,17	0,50	0,59	70
0,14	0,37	0,55	80
0,12	0,28	0,51	90
0,10	0,22	0,47	100