The influence of specific type of instability over structure formations in accretion discs

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Abstract

When we investigate the different structures in accretion flows, we find out the relation between arising of any type of instability and the engines of structure formations, when all required conditions exist.

Here we present how some of the instabilities effect to the process of the structures appearing.

Magnetohydrodynamical instabilities

a) Balbus-Hawley instability.

There are places in a hydrodynamical flows, where the velocity field changes strong (the shock fronts). In thees places, because of the differential rotation of the parts of the flow with the big different in density and velocity, there are arised the conditions for magnetic shear instability, which is known as Balbus - Hawley instability. In the presence of magnetic field, there is a destabilization effect of the differentional rotating flows and this instability is generating mechanism of the turbulence in the flow [7].

b) Kelvin - Helmholtz instability.

This instability acts on the boundary of two fluid flows, which in our case would be two parts of the accretion flow.

If we disturbe weak that boundary, the velocities increase and at the different densities it yields the folowing condition of instability:

$$\rho_1\rho_2\left((v_1 - v_2)k\right)^2 \leq (\rho_1 + \rho_2)(\rho_1 - \rho_2)k_\lambda g \tag{1}$$

where v_1 and v_2 the velocities of two flows
and ρ_1 , ρ_2 are their densities.

The relation between frequency ω and wave vector k_λ is given with the disspersion relation [4]:

$$\frac{\omega}{k} = \frac{\rho_1 v_1 + \rho_2 v_2}{\rho_1 + \rho_2} \pm \left[\frac{g}{k}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right) - \frac{\rho_1\rho_2(v_1 - v_2)^2}{(v_1 + v_2)^2}\right]^{1/2} \tag{2}$$

The Kelvin - Helmholtz instability arise if the expression in the square brackets of the above relation is negative and there is a difference in the two flows velocities. In the presence of this instability there are formed an undulations in the boundary and further this yields to vorticity formation.

Other instabilities

The appearing of structures in accretion discs is the result not only of the magnetohydrodynamical instability. In the fluid media of accretion disc it is consist the conditions for other type of instabilities.

a) Turing instability.

In some systems the coupling between two transport procceses provides the engine of instability. Then the growth of this instability is defined by the difference of the diffusion coefficients in the different direction of the transport acting there [1].

The diffusion coefficient presents in the reaction - diffusion equation. At first we apply one standard form of this equation [2]:

$$(2) \qquad \frac{\partial C}{\partial t} = F(C) + D\nabla^2 C$$

where the first term in the right side is the reaction and the second is the diffusion. D is the diffusion coefficient (or matrix of the transport coefficient), C- a concentration of matter.

The reaction - diffusion systems are the exibition of spatial or temporal patterns if they are far from equilibrium [2], which is one an important condition to forming the dissipative structures [8]. The key role in all application of the reaction-diffusion eqaution as partial differential eqation, is this simple combination of reaction with diffusion in the right handside of the eqaution.

Than envisaging the condisions of Turing instability, we may write the eqaution in the form:

$$(3) \qquad \frac{\partial C}{\partial t} = F(C) + D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2}$$

That difference between two components of diffusion coefficients is the necessary restriction to appear the Turing instability [2].

Now to reach of the deduction of this assertion we present the following eqautions and their transformations.

In the first place this is the continuity equation, which express the mass conservation:

$$(4) \qquad \frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0$$

The conservation of momentum for each gas element gives with Euler equation:

$$(5) \qquad \rho\frac{\partial v}{\partial t} + \rho v.\nabla v = -\nabla P + F$$

where for both equations the quantities ρ, v, P, F are respectively: the density, the velocity, the pressure and the selected force.

We present the equation of motion for viscous fluid (Navier Stokes eq.) in cylindrical coordinates. Because of the averaging in z-direction we express all derivatives in the terms of the coordinates (r, φ) :

$$\begin{gathered}\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi}-\frac{{V_\varphi}^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\frac{1}{\rho}F_r+\\ \nu\left(\frac{\partial^2 V_r}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \varphi^2}+\frac{1}{r}\frac{\partial V_r}{\partial r}-\frac{2}{r^2}\frac{\partial V_\varphi}{\partial \varphi}-\frac{V_r}{r^2}\right)\end{gathered}\tag{6}$$

$$\begin{gathered}\frac{\partial V_\varphi}{\partial t}+V_r\frac{\partial V_\varphi}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi}-\frac{V_r\,V_\varphi}{r}=-\frac{1}{\rho r}\frac{\partial P}{\partial \varphi}+\frac{1}{\rho}F_\varphi+\\ \nu\left(\frac{\partial^2 V_\varphi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 V_\varphi}{\partial \varphi^2}+\frac{1}{r}\frac{\partial V_\varphi}{\partial r}-\frac{2}{r^2}\frac{\partial V_r}{\partial \varphi}-\frac{V_\varphi}{r^2}\right)\end{gathered}\tag{7}$$

Here ν is the kinematic viscousity, V_r and V_φ are the two component of the velocity.

The equation of energy transfer could be presents as follows:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+\rho\varepsilon\right)+\left[\left(\frac{1}{2}\rho v^2+\rho\varepsilon+P\right)v\right]=f.v-\nabla.F_{rad}\tag{9}$$

where $\frac{1}{2}\rho v^2$ - the kinetic energy per unit volume,

$\rho\varepsilon$ - internal or thermal energy per unit volume.

The last term in a square brackets represents the so-called pressure work.

On the righ hand side:

F_{rad} - the radiative flux vector;

$-\nabla.F_{rad}$ - gives the rate at which radiant energy is being lost by emission, or increased by absorbtion.

In accretion disc we consider the transport of "vortical" function or vorticity, which we may imply with Ψ. This term we take from the vortical equation [5]:

$$\left(\frac{\partial}{\partial t}+\vec{v}.\nabla\right)\frac{\nabla\times\vec{v}}{\rho}=0\tag{10}$$

which is received, combining the rotation of momentum equation and the continuity equation. So, $\Psi=\nabla\times\vec{v}$ and for eq. (10) yelds:

$$\left[\frac{\partial\Psi}{\partial t}+\nabla.(\Psi\vec{v})\right]\frac{1}{\rho}=f\tag{11}$$

We express that equation in cylindrical coordinates in the terms of the r,φ again:

(12) $$\left[\frac{\partial \Psi_r}{\partial t} + V_r \frac{\partial \Psi_r}{\partial r} + \frac{\Psi_\varphi}{r}\frac{\partial V_r}{\partial \varphi} - \frac{\Psi_\varphi{}^2}{r}\right]\frac{1}{\rho} = f$$

(13) $$\left[\frac{\partial \Psi_\varphi}{\partial t} + V_r \frac{\partial \Psi_\varphi}{\partial r} + \frac{\Psi_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi} - \frac{\Psi_r \Psi_\varphi}{r}\right]\frac{1}{\rho} = f$$

Here with f we express the transport engine of the vortex or that is the diffusion from eq. (3) and for our considering the f is in the form: $D\nabla^2\Psi$.

Taking in account the vortical equation (11) and the expressions (12) and (13), the reaction - diffusion equation (3) becomes [8]:

(14) $$\frac{\partial \Psi_r}{\partial t} = h(r,\varphi) + D_r \nabla^2 \Psi_r$$

(15) $$\frac{\partial \Psi_\varphi}{\partial t} = g(r,\varphi) + D_\varphi \nabla^2 \Psi_\varphi$$

h and g are the source functions and they take the form: $(\Psi.\nabla)v$

Thereby we obtained two equations with different diffusion coeficients, which is expanded in two components.

The evidence that in accretion discs the neccesary condition the ratio between D_r and D_φ to be not equal of unity and zero exists. That gives us the confirmation of possibility to appearing of Turing instability in this reaction - diffusion system.

Since, these instabilities are the expression of spatial pattern of the biffurcation's area, in consequence of them the structures may arized in the disc, particularly: vortical structures and so calls the Rossby solitons.

What is in actually the Rossby vorticities and which other instabilities give rise to them.

b) Rossby instability.

When we consider nonmagnetized Keplerian accretion disc as a result of nonaxysimmetric perturbations, it rised instability which cause Rossby vortices in nonlinear limit. The presence of such vortices would be crucial for the hydrodynamical transport of angular momentum in accretion discs [6]. A wave of nonlinear Rossby vortices carries the mass and entropy maximum inward, exciting further vortices which transport the angular momentum outward.

For expression of based nonbarotropic disc eqautions, we use the cylindrical system of coordinate again. We consider surface density $\Sigma(r) = \int_{-h}^{h} dz\rho(r,z)$ and vertically integrated pressure $P(r) = \int_{-h}^{h} dzp(r,z)$.

The perturbed quantities of the surface density, pressure and velocity are expressed as follows:

$$(16)\qquad \begin{aligned} \widetilde{\Sigma} &= \Sigma + \delta\Sigma(r, \phi, t) \\ \widetilde{P} &= P + \delta P(r, \phi . t) \\ \widetilde{v} &= v + \delta v(r, \phi, t) \end{aligned}$$

Then the equations for the perturbed disc are:

$$(17)\qquad \begin{aligned} &\frac{D\widetilde{\Sigma}}{Dt} + \widetilde{\Sigma}\nabla.\widetilde{v} = 0 \\ &\frac{D\widetilde{v}}{Dt} = -\frac{1}{\widetilde{\Sigma}}\nabla\widetilde{P} - \nabla\Phi \\ &\frac{D}{Dt}\left(\frac{\widetilde{P}}{\widetilde{\Sigma}^{\Gamma}}\right) = 0 \end{aligned}$$

where $D/Dt = \partial/\partial t + v.\nabla$ and $S = P/\Sigma^{\Gamma}$ is the entropy of the disc matter. Here the last eq.( ) shows the isentropic behavior of the disc matter.

Since this instability is related to the entropy behavior, it is yield, so call, key function $\Re(r) = \Lambda(r) S^{2/\Gamma}(r)$, which has a maximum or minimum. Than the instability is possible only if $\ln\left(\Lambda S^{2/\Gamma}\right)$ vanishes at some r.

Here in the way described above, we obtaine the vortical equation again in the form:

$$\frac{D}{Dt}\left(\frac{\Psi_z}{\Sigma}\right) = \frac{\nabla\Sigma \times \nabla P}{\Sigma^3}, \text{ where } \Psi_z = \hat{z}.\nabla \times v \text{ is the vorticity.}$$

For a barotropic flow the right hand side of the equation become zero and each fluid element conserves its specific vorticity.

In the contrary case the term $\nabla\Sigma \times \nabla P \propto \nabla T \times \nabla S$ destroys this conservation and give possibility of the pressure force to produce vorticities in the flow.

Discussion

Here we haven't mentioned the all kind of instabilities, which act in accretion flow in general. Our aim was to show their reference to structures formation in accretion discs. We proved this with our analiticaly computation on the base of accretion disc equations and the expressions of concrete instability.

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