THE GRAVITY DARKENING EFFECT: FROM VON ZEIPEL UP TO DATE

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Abstract

The gravity darkening effect and the theoretical methods developed so far – starting from von Zeipel, who theoretically predicted it– are briefly referred. The various approaches made by several investigators to get the gravity darkening exponents of close binaries, from the analysis of their observational data, are also referred and discussed. Special attention is given to the latest obtained results.

1. Introduction

The existence of the gravity darkening effect over the surfaces of stars in radiative and hydrostatic equilibrium, distorted by axial rotation, was theoretically predicted as early as 1924 [1]. While a few years later it was extended to tidally distorted configurations [2].
On the other hand, the photometric consequences of this effect on the light changes of close eclipsing binaries for centrally condensed stars in total or partial radiation were studied [3,4]. Moreover, the laws of gravity darkening for the black body case -appropriate for any particular effective wavelength- were formulated [5].

Besides the forgoing mentioned very early approaches to the subject, the effect of gravity darkening has attracted the interest of many investigators. So, it was theoretically studied for stars with convective envelopes, too [6]; and recently it became the subject of extended examination [7,8,9,10].
Moreover, attempts were made to get gravity darkening values via observational data analysis, using various techniques. These were mainly applied to close binary stars, where the effect was expected to be stronger because of their components' distortion.
Thus, in this brief review, the most important of the theoretical as well as the observational attempts to study the gravity darkening effect will be referred and discussed.

2. Theoretical Approaches

More than a century ago, in 1924, von Zeipel [1] concluded that: "*the emergent flux, **F**, of the total radiation at any point over the surface of a rotationally or tidally distorted star in radiative equilibrium varies proportionally to the local gravity **g***". That is: $F \prec g$ or $F \prec g^{\tau}$, where the exponent τ is known as the *gravity-darkening coefficient*. And if the energy transfer in the sub-surface layers of a star is purely radiative, it was found that τ is equal to unity ($\tau=1$).

Many years later, a different value for τ was given. Actually, fitting together convective envelopes having different surface gravities at a depth where the temperature gradient becomes adiabatic, it was found that $\tau=0.32$ [6].
And since between the emergent flux and the temperature there exist the relation:
$F \sim (T_{eff})^4$, it is obviously: $T_{eff} \prec g^{\tau/4}$ or: $T_{eff} \prec g^{\beta}$
where β -known as the *gravity darkening exponent*- is equal to $\tau/4$.
The relations: $F \prec g^{\tau}$ or $T_{eff} \prec g^{\beta}$ are also referred and known as the gravity darkening law, and:
a) $\beta=0.25$, when the energy transfer is purely radiative, (von Zeipel's law), while
b) $\beta= 0.08$, for stars with convective envelopes (Lucy's law).

Later, from more detail studies, it was pointed out that the von Zeipel gravity-darkening law is strictly only true [11]. That is, if: 1) the rotation law is conservative, i.e. the centrifugal force is derivable from a potential and 2) the radiative flux is approximated by the diffusion equation $F=-\frac{4\pi}{3}\frac{1}{k\rho}\nabla B$ used in stellar interiors. All symbols used in the previous equation have their usual meaning: k & ρ stand for opacity and density, respectively; while, B is the integrated Planck function; that is: $B=\frac{1}{\pi}\sigma T^4$.
Then, ρ, T, and the pressure P are all functions of the total potential Ψ (gravitational plus centrifugal), and: $F=-\frac{4\pi}{3}\frac{1}{k\rho}\frac{dB}{d\Psi}\nabla\Psi=-f(\Psi)g$, where $g=\nabla\Psi$ is the effective gravity.

If the transfer equation is used, but meridional circulation is neglected, a perturbation treatment shows that the emergent flux F satisfies the relation: $|F| \propto g^{1/2}$ for all conservative rotation laws. The inclusion of circulation, however, seems to restore von Zeipel's law, $|F| \prec g$. For non-conservative rotation laws, $|F|$ is no longer a simple power of *g*, even assuming the diffusion equation.

Similarly, for magnetic stars, it was concluded that the important factor determining ***β*** might be **the ratio of magnetic energy to rotational energy** rather than the absolute value of either [12]. And for most of the models studied, a good approximation for the gravity darkening would be: $| F | \propto | g |^{1.1}$. However, there is some evidence that slowly rotating, strongly magnetic stars could have a much stronger dependence of $| F |$ on $|g|$, comparable to that found by others [11], for some non-magnetic rotating models [13]. Generally we could say that for most of the models studied, there is only a small departure from von Zeipel relation. However, for stars with a large ratio of magnetic to rotational energy the variation of emergent flux with gravity could be quite strong, (which could be relevant for a few ***Ap*** stars).

On the other hand, for stars with convective envelopes, it was found that assuming a simple opacity law, and ***if the entropy is constant*** in an envelope, the values of ***β*** are compatible with the earlier derived [14]. While, a modified form of Lucy's gravity darkening law was obtained and it was shown that, for the secondary components of ***W UMa*** type contact binaries, the gravity-darkening coefficient was ***τ≈0.32***, in agreement with Lucy's predictions [15]. In particular, the gravity darkening exponent ***β*** was calculated for a broad spectrum of masses and mass ratios, and was found that for the secondary component in ***W UMa*** type stars ***β~0.08***. This means that ***β*** **does not change significantly** with the mass-ratio ***q*** and the mass $\boldsymbol{m_s}$ of the secondary component. Similarly, for the secondary main sequence component in ***cataclysmic variables*** the gravity-darkening exponent does not change with mass. Actually: for $M_2 \leq 0.7M_{\odot}$, $\beta \approx 0.05$, while for $M_2 \geq M_{\odot}$, $\beta \approx 0.08$ [15].

In the mean time, it had been pointed out that for a contact binary in hydrostatic equilibrium having a common convective envelope, ***the convective flux depends on the effective temperature only and not on its gradient***. In other words: ***there is no relation between T_{eff} and effective gravity*** [16]. So ***β=0***, but this approach, applied to the reflection effect, predicts ***zero albedo***, which disagrees with observations.

This was the situation till 1980 or even 1990. But, the gravity darkening effect has attracted the interest of many investigators, as already mentioned. So, recently (1994), the very high values of ***β*** obtained for some semi-detached binary systems [17] were interpreted as an enthalpy transport linked with the mass exchange process [18]. And just a few years ago (1997-1999), the gravity darkening (or brightening, as is also referred by some investigators) in non-illuminated convective gray and non-gray atmospheres for different T_{eff}, in the range: $3700<T_{eff}< 7000$ was studied [7,8]. It was then found that ***β*** depends on T_{eff}, being rather insensitive to variations of mixing length parameter, of the stellar mass and to the use of gray or non-gray atmospheres. These results, confirmed Lucy's [6] value ***β=0.08***, for $T_{eff} \approx 6500$ K.

The latest studies on the subject have been presented by Claret -1998 & 2000- in two papers [9,10]. In the first, the gravity-darkening exponent was presented as a function of mass and age, based on the triangles strategy [19], and the adopted stellar models [20] had a representative chemical composition of X=0.70 & Z=0.02. The conclusion was that: The old values of the gravity darkening exponent ***β=0.25 and β=0.08*** for radiative and convective envelopes, respectively are superseded by new results according to which a ***smooth transition*** is achieved between the ***two extreme energy transport mechanisms***. In other words: ***The two processes of energy transport can exist even simultaneously in a determined stellar envelope.***

In the second paper [10], it was pointed out that the light we observe from the components of a binary system depends not only on the geometrical configuration but also on how the specific intensities are distributed along the stellar disk. Often, such components are distorted by rotation and tides, and the flux distribution is not uniform over their surfaces. Thus: *The gravity-darkening phenomenon is related not only with atmospheric parameters but also with the internal stellar structure and with details of the rotating law.* Moreover, the influence of changing the input physics on the gravity-darkening exponents was investigated, and found that they depend slightly on the chemical composition mainly in the zone of the radiative/convective phase transition. For deep convective envelopes, it is found no significant differences in ***β's*** computed for different mixing-length parameter.

Finally, the gravity distribution on the outer equipotential surface of contact binaries has been presented in three dimensions [21]. In particular, the ***iso-g*** curves were computed and plotted on the outer surface ***Cs*** of contact configurations for various combinations of the filling factor (***f***), and the mass ratio (***q***). While, applications were later made to specific systems [22,23].

3. Observational Treatments

Gravity-darkening coefficients -as second order parameters- is very difficult to be determined directly from the observations. However, several approaches were made -by various investigators, using different methods- to derive such parameters. The results of these efforts considering either one particular star, or more, were published in one or in a series of papers. And it is worthwhile to refer a few details concerning some of the derived results, since in some cases they are not consistent with the theoretical findings.

So, for contact binaries:

- From the analysis of **26** late type ***W UMa***-type binaries, the gravity-darkening coefficient, ***(τ=4β)***, was found to be greater than unity [24]. In particular it was in the range **(1.1-7.1)** with a typical value around ***2***.
- A similar result was found for the contact binary ***RZ Com***[25] for which ***τ*** seemed to be from **1.1** to **1.5**, instead of **0.32** appropriate for stars with convective envelopes.
- In general, values of ***τ*** or ***β*** greater than the theoretical ones have been reported for contact systems of ***W UMa-type***. To be more specific: it was found to be so for ***TX Cnc*** [26], for ***AK Her*** [27], and for **16** other analyzed systems [28].
- The gravity darkening exponent ***β*** was found to be equal to zero for **4** *late-type* contact binaries, and under certain conditions [29].

Concerning the semi-detached systems, very different results have obtained by the various investigators. Thus:

- Very high values of ***τ*** have been reported for the secondary components of *Algol-type* binaries [30]. To be more specific: The analyzed sample contained stars with convective and radiative components. A high dependence of the surface brightness with the gravity was found. The derived ***τ*** values were in the range **2.3 to 9.4**, which is in disagreement with the classical theoretical values.
- High values of ***τ*** were also derived for the primary components of *reverse Algols* [17]. Although in general they were found to be less than for *Algols*.
- Rather normal gravity darkening values, and in very good agreement with theory, derived from the analysis of some semi-detached systems [31,32].

4. Summary - Discussion

Since the pioneer work of von Zeipel and the gravity darkening law he found for stars in radiative equilibrium: $F \prec g^{\tau}$ or $T_{eff} \prec g^{\beta}$, a lot has been added to our knowledge. So, while von Zeipel had considered rotationally distorted stars, the study was extended to tidally distorted configurations [2]. In general it was found that $\tau=1$ or $\beta=0.25$ for stars in radiative equilibrium. Moreover, the conditions under which this law is valid and how it varies especially for magnetic stars were examined, too [11,12].

Then, stars with convective envelopes were studied [6], and a different value for the gravity darkening coefficient (or the exponent) was theoretically derived: $\tau=0.32$ or $\beta=0.08$.

And recently it was suggested that both energy transfer mechanisms could be simultaneously in action; so, all intermediate values of τ or β should be expected [9,10].

Besides, the gravity distribution over the surfaces of contact systems has been presented in 3-D graphs [21], and applied to particular binaries [22,23].

As concerns the efforts to get gravity darkening exponents from the analysis of observational data, most of the firstly analyzed systems were contact binaries of ***W UMa***-type. Apart from them, the best objects for which theoretical gravity darkening could be tested by observations were the distorted components of semi-detached systems. So, the light curves of such systems were analyzed using different methods and techniques.

One of the original and best such attempts was that in which systems of all kinds with both radiative and convective envelopes were included. The obtained mean values of τ were **0.91** and **0.31**, respectively [33], in quite good agreement with the theory. The same is true in some other cases, too. But, as was already referred, the obtained results from such analyses were not always in close accordance to the theoretical predictions. Why is this so? Does it have a special meaning?

Regarding contact binaries:

From a quantitative analysis, made in 1968, it was concluded that either the gravity darkening is considerably larger than that predicted by the theory, or the only effective way of reconciling the theory with the observations would be to assume that the mean fractional

radii of the two components are appreciably larger than those appropriate for contact models in which the two stars just fill the largest closed Roche equipotentials capable to contain their mass [24].

In general, values of τ or β greater than the theoretical ones have been reported for contact systems of ***W UMa-type***. And this does not concerns only the systems ***TX Cnc***, ***AK Her***, or some others previously referred [26,27,28]. A similar situation has reported for some other cases, too [34,35]; but the obtained gravity values although greater than for purely convective envelopes are consistent, at least for some cases, with the latest theoretical findings [9,10]. On the other hand, it was found that curiously there is no evidence of anomalous large gravity darkening values in early type close binary systems, and it was suggested that there might exist some non-linear peculiar effect to contact systems with common convective envelopes [36].

Moreover, only for **4** contact systems, namely ***W UMa*** (itself), ***XY Leo***, ***TX Cnc*** and ***AH Vir***, the gravity darkening exponent β was found to be very small, almost zero [29]. This is in accordance to the only one theoretical work that differs from all others [16]. But if this is so, it was reported that the degree of contact had to be greater (of the order of **25%**), instead of **15%** if the theoretical value of $\beta=0.08$ was adopted, according to Lucy law for stars with convective envelopes. Moreover, in the same work [29] it is reported that there is some indication that the discrepancy between photometric and spectroscopic mass-ratios is reduced the value $\beta=0.00$ is used.

As regards semi-detached systems of *Algol* or *reverse Algol* type both normal and higher gravity-darkening values have been reported. To be more specific:

From the analysis of the infrared light-curves of ***Algol*** (itself, at $\lambda=1.6\mu$) [37] and it was found that the monochromatic coefficient τ_λ was **3** to **4** times larger than that resulting from the theory.

Similarly, the calculated gravity darkening values for the primary components of the *reverse Algols* -although distinctly smaller than those derived for the secondary components in semi-detached systems of normal *Algols* [29,17]- in both cases were found to be higher than the theoretical ones. Based on the mass-out-flow darkening model, these results would indicate that the rates of mass transfer in *reverse Algols* are not so high, and might imply that the systems are not in the rapid phase of mass transfer [31].

And although recently the very high values of β obtained for some semi-detached binary systems [17] were interpreted as an enthalpy transport linked with the mass exchange process [18], this might be ***simply the result of the analysis method used***. This may be really the case, since these values are not supported by the recent simultaneous ***uvby*** observations for the ***VV UMa*** case [10], as well as for some other recently re-analyzed systems [31,32]. It is very important and special attention has to be paid to the analysis method used, which has to be checked before applied to real stars [32]. If the method is not good the obtained results, either being close to the theory [38] or not being consistent with it [17, 30], might not be reliable.

Two other important subjects to consider during the light curves analysis is the spot activity, since both ***W UMa-type*** and ***Algol-type*** binaries show this kind of activity due to their magnetic fields, and the third light. The latter has not been examined in detail. As concerns the first, for two semi-detached systems, namely ***LT Her*** & ***TV Cas***, it was found that if a spotted model is used [32] the gravity darkening exponents are consistent with the theory. Although some of the analyzed contact systems show such an activity, it has not be connected with possible abnormal values of the gravity darkening.

It is again emphasized that great attention has to be paid to the analysis method used, as well as to the specific characteristics of the chosen systems. For example some discrepancies from the expected gravity darkening values seems to exist for those systems in which both of their components are very hot [32]. This is not so, for "normal" semi-detached systems [39].

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