S E N S ' 2 0 0 6
Second Scientific Conference with International Participation
SPACE, ECOLOGY, NANOTECHNOLOGY, SAFETY
14 – 16 June 2006, Varna, Bulgaria

THE PARAMETRIC INSTABILITY OF OSCILLATOR IN THE CASE OF COMPLEX MODULATION. THE EXACT SOLUTION MODEL

N .N. Erokhin[1] , N.S. Erokhin[1)], Vladimir Damgov[2)]

[1)] *Space Research Institute of RAS, Moscow, Russia*
e-mail: nerokhin@mx.iki.rssi.ru
[2)] *Space Research Institute of BAS, Sofia, Bulgaria*

Keywords: oscillator, parametric instability, exact solution.

Abstract.
It is considered the exact solvable mathematical model of oscillator parametric instability for the case of complex modulation function. The analytical solution of this model is presented and numerical calculations are performed to study the instability parameters dependence on the modulation function spectrum. The results obtained are of the great interest for nonlinear processes in the space plasma, the astrophysics, for planet researches and the biophysics also including cases of stochastic resonances.

Introduction

The studying of parametric instability dynamics is of the great interest for a number of applications, in particular, for the electromagnetic radiation generation, in the analysis of powerful electromagnetic waves interaction with a plasmas including the waves absorption in resonance layers and nonlinear reflection effects, for the interpretation of observational data related to alfven waves in the solar wind and so on (see, for example, [1,2]). In the present paper using the approach developed early in [3] for linear oscillator model we consider the dynamics of parametric instability at the main resonance in the dependence on the type of function describing the modulation of oscillator frequency. The exact solution of this problem is presented by analytically with a some arbitrary function. Then by the numerical calculations of this solution it was obtained the temporal profile of instability growth rate, the amplification coefficient of oscillation amplitude, the threshold of instability saturation under the variable level of oscillator modulation and other characteristics in the dependence on spectrum of modulation function and the phase relations of its harmonics. It is to be a matter of principle that the problem exact solution includes an arbitrary function allowing to change significantly the scenario of oscillation generation

It has been shown the possibility to control the dynamics of parametric instability development by the corresponding choice of oscillator frequency modulation function. It is considered the parametric instability dynamics in the case of modulation function including beyond the regular part additionally the random component. The results obtained are of the great interest for the radiophysics and the nuclear fusion researches, in the analysis of nonlinear wave processes in the space plasma. It may be useful for the astrophysics, the biophysics, for planet studyings and in the stochastic resonance investigations

Basic equations and their solution analysis

The simple physico-mathematical model of parametric instability is described by the oscillator equation like this

$$d^2 x / dt^2 + \omega_0^2 \cdot [\, 1 + Q(t) \,]\, x = 0. \qquad (1)$$

Here the function Q(t), so-called the modulation function, represents the variation of oscillator frequency in square. Let us introduce the nondimensional time $\tau = \omega_0 \cdot t$. The solution of equation (1) may be written by the following expression

$$x_1(\tau) = x(0) \cdot \exp [\, W(\tau) \,] \cdot \cos \tau, \qquad (2)$$

$$W(\tau) = \int_0^{\tau} g(\tau) \cdot \cos \tau \cdot d\tau, \qquad (3)$$

where $g(\tau)$ is an arbitrary function. So the modulation function $Q(\tau)$ is determined now by

$$Q(\tau) = 3 \cdot g \cdot \sin \tau - g^2 \cdot \cos^2 \tau - (dg / d\tau) \cdot \cos \tau, \qquad (4)$$

(see for details the monography [3]). Let us to consider firstly the case of monochromatic modulation at the modulation frequency close to the double oscillator frequency when we take $g(\tau) = \varepsilon \cdot \cos (1 + \mu)\tau$ with frequency detuning μ to be small $\mu \ll 1$. Then using (3), (4) for functions $W(\tau)$, $Q(\tau)$ the following formulae are obtained

$$W(\tau) = (\varepsilon / 2\mu) \cdot \sin \mu\tau + [\varepsilon / 2 \cdot (2 + \mu)] \sin (2 + \mu)\tau, \quad Q(\tau) = 3\varepsilon \cdot \sin \tau \cdot \cos (1 + \mu)\tau +$$
$$+ \varepsilon \cdot (1 + \mu) \cdot \cos \tau \cdot \sin (1 + \mu)\tau - 0{,}25\, \varepsilon^2 \cdot [\, \cos \mu\tau + \cos (2 + \mu)\tau \,]^2 . \qquad (5)$$

According to formulae (5), in this case the low-frequency (with the frequency μ) modulation of oscillation amplitude on the main oscillator frequency ω_0 is occurred. The maximum of oscillation amplitude amplification is given by $\eta = \exp(\varepsilon / 2\mu)$ and it is large for the small frequency detuning $\mu \ll \varepsilon / 2$. The typical amplification time is of the order of $\Delta\tau \sim \pi / 2\mu$. The plot of modulation function $Q(\tau)$ is presented on the fig.1a for the case $\varepsilon = 0{,}2$, $\mu = 0{,}1$.

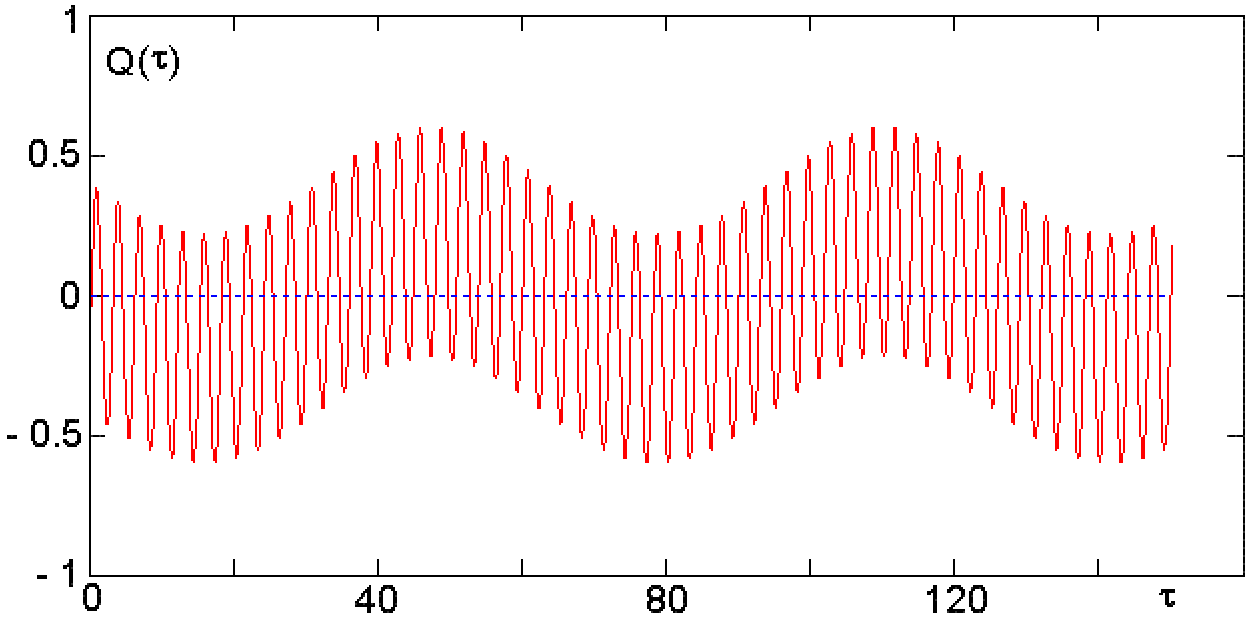

Fig.1a. The plot of modulation function.

According to fig.1a there is the fast (with the $2\omega_0$ frequency) modulation of oscillator frequency in square. Moreover the slow envelope of the level modulations observed with the period $T = 2\pi/\mu$. The function $W(\tau)$, determining the oscillation amplification at the main resonance, is given on the fig.1b for the case of parameter choice ε = 0,2, μ = 0,01, when max W(t) = 10. The maximum value of oscillation amplitude amplification is very large $2.2 \cdot 10^4$. It is necessary now to note the important physical circumstance that parametric instability considered is the reversible one in its nature.

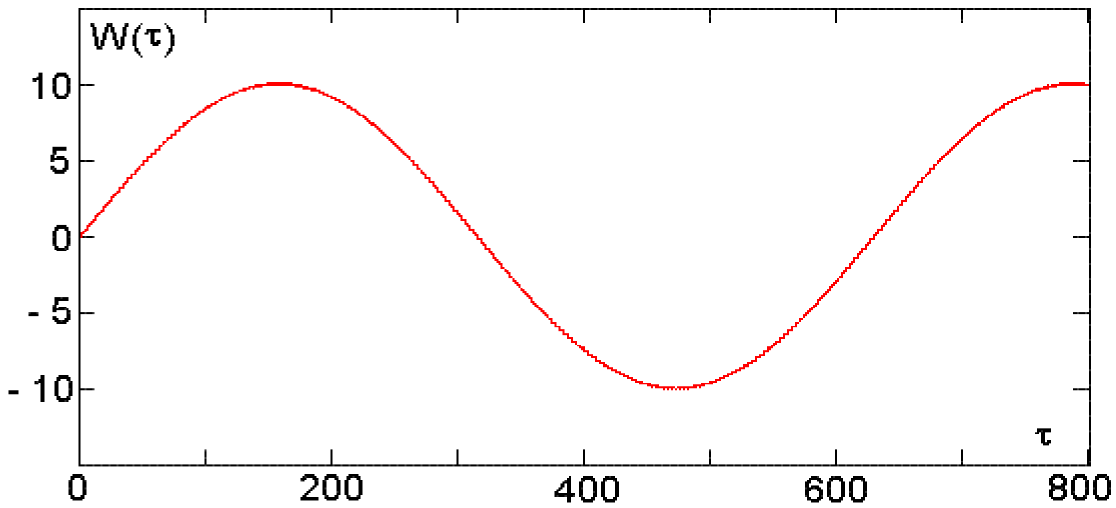

Fig.1b. The amplification function plot.

It is of the great interst to choose the function $g(\tau)$ like this $g(t) = b\cos(t + \theta)$ with a some phase θ. In the fig.2 results of numerical calculations are presented. For the comparison purpose two versions of function $g(\tau)$ choice were calculated namely $g_1(t) = b\cos(t + \theta_1)$, $g_2(t) = b\cos(t + \theta_2)$ with the equal amplitudes b = 0.08 but the different phases $\theta_1 = 0$ and $\theta_2 = 1.43$. According to the fig.2a the modulation levels in both cases are ptactically the same. Nevertheless (see the fig.2b) the amplification functions $W(\tau)$ differ drastically. In the second case the instability growth rate is less 7 *times ! Moreover, for the phase choice $\theta = \pi / 2$ the instability growth rate is equal zero.*

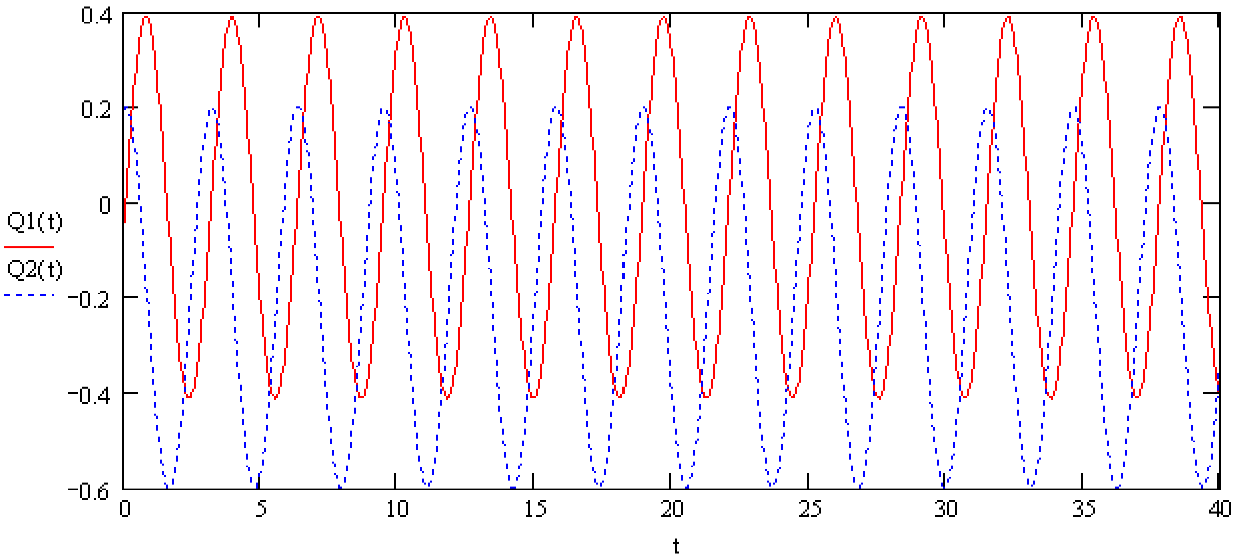

Fig.2a. The plots of modulation functions $Q_1(t)$, $Q_2(t)$.

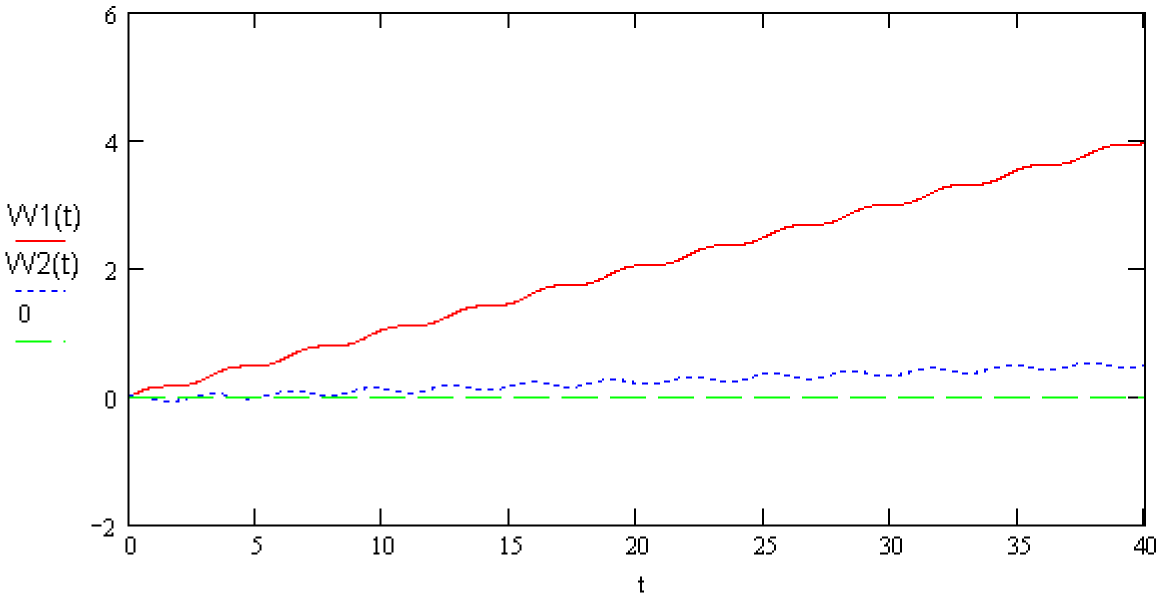

Fig.2b. The plots of amplification functions $W_1(t)$, $W_2(t)$.

It is of interest to consider the parametric instability for the case of function g(t) to be the sum of harmonics with the close amplitudes but having different phases and frequency detunings. The calculations were performed for the case

$$g(t) = b \cdot [\cos(t + \mu_1 \cdot t + \theta_1) + \varepsilon_2 \cdot \cos(t + \mu_2 \cdot t + \theta_2) + \varepsilon_2 \cdot \cos(t - \mu_2 \cdot t + \theta_3)]$$

with parameters $b = 0.08$, $\mu_1 = 0$, $\mu_2 = 0.061$, $\theta_1 = 0$, $\theta_2 = 1.47$, $\theta_3 = -0.97$, $\varepsilon_2 = 0.91$. According to calculations for the close harmonic amplitudes at the large times the main contribution to parametric amplification is provided by the harmonic having the zero frequency detuning $b \cdot \cos(t + \mu_1 \cdot t + \theta_1)$, $\mu_1 = 0$.

It was considered also the case of function g(t), containing 11 harmonics with eqial amplitudes and small frequency detunigs namely the governing function g(t) was taken like this $g(t) = \sum_n a_n \cdot \cos(\theta_n + t + \mu_n \cdot t)$, where $\mu_n = \beta + \delta \cdot (n - 6)$, $a_n =$

b , n = 1, 2 …11 and β = 0.002 , δ = 0.007 but θ_n were random phases. The plott of resonance amplification functions $W_{13}(t)$, $W_2(t)$, related to different harmonic groups respectively n = 1, 2, 3, 9, 10, 11 and n = 4, 5, 6, 7, 8 , are given in fig.3a.

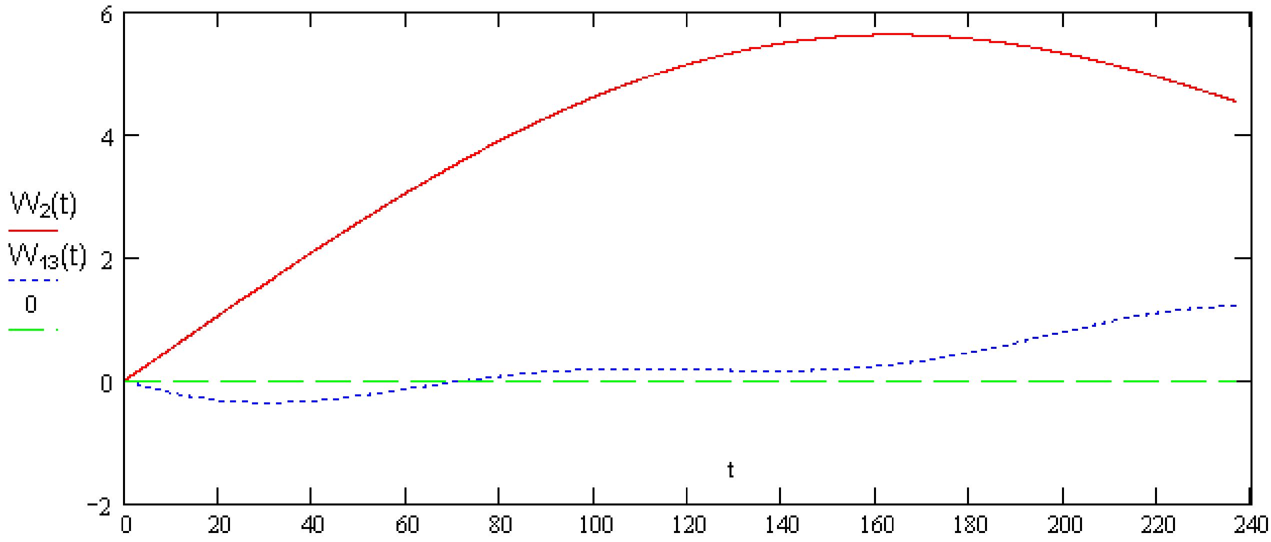

Fig.3a.The amplification functions plots for different harmonic groups.

It is seen that the main amplification of oscillation amplitude is conditioned by the most close to the parametric resonance harmonics with numbers n = 4, 5, 6, 7, 8. Due to the small but finite frequency detuning of the harmonics with n = 6, the oscillation amplification is saturated at the large enough times t ~ 200 when max $W(t) \approx 6.14$. So the amplitude amplification coefficient exp(W_{max}) is equal 464. The plot of modulation function Q(t) is given on the fig.3b.

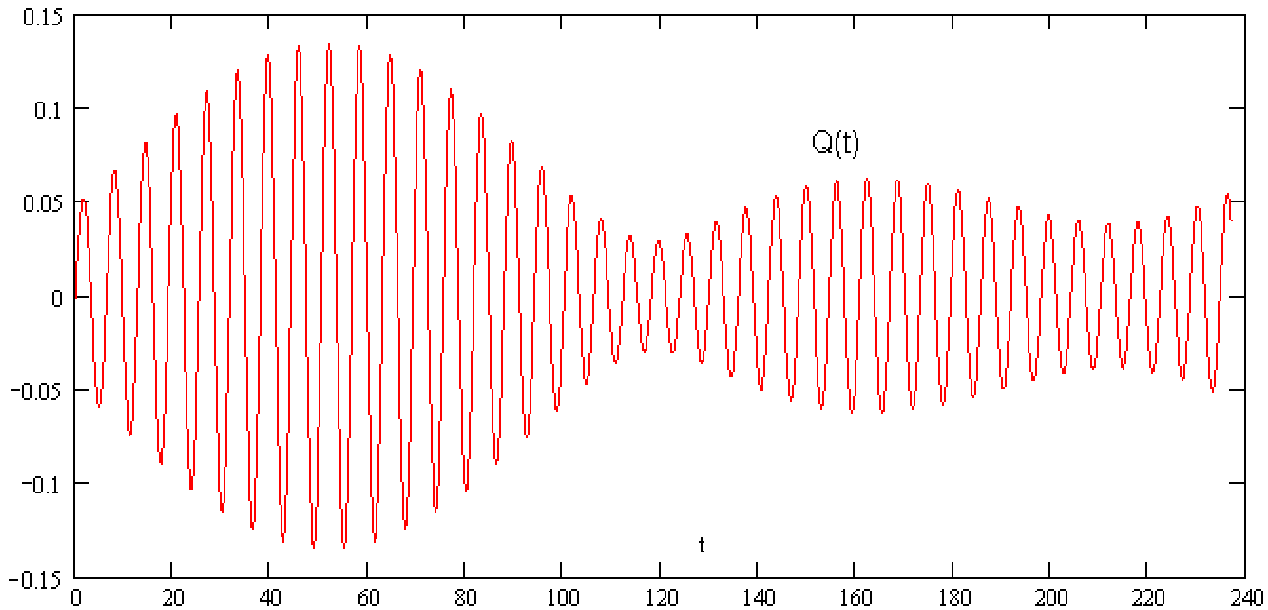

Fig.3b.

The special case β = 0, θ_6 = 0 when the harmonic n = 6 has the exact resonance with maximum contribution to the instability growth rate is shown in the fig.3c by the plot of full amplification function W(t). According to fig.3c the instability saturation is absent because the function W(t) is increased unlimitedly with time *growth.*

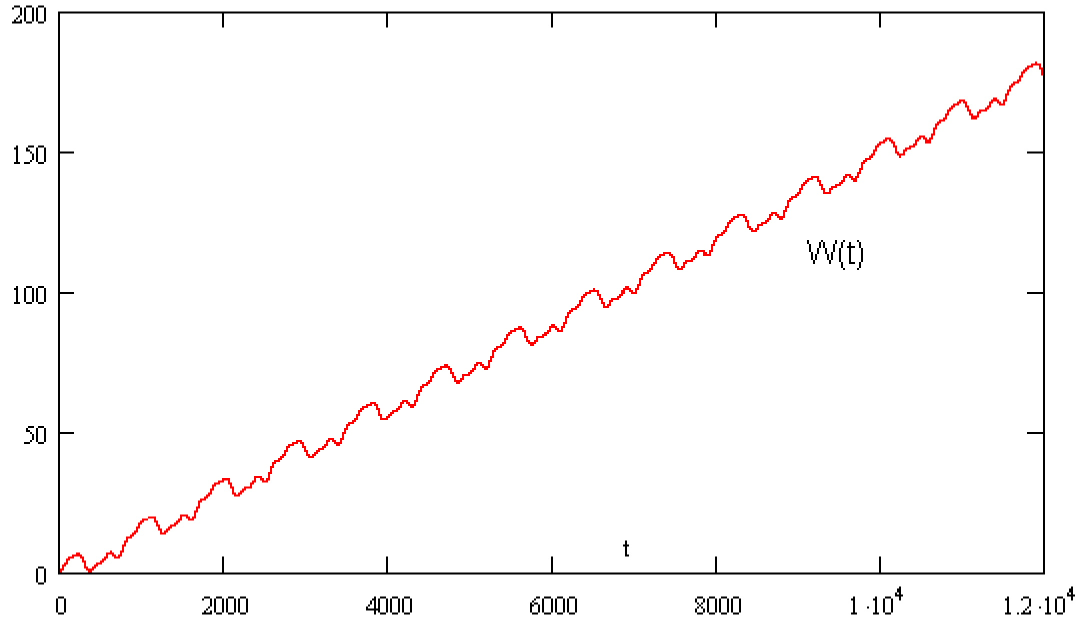

Fig.3c. The plot of amplification function W(t) in the case of presence in modulation spectrum the harmonic to have the exact parametric resonance.

Conclusions

The results of analysis performed may be formulated as the following. Firstly, on the basis of exact solution the generation of oscillations is studied during the parametric instability of oscillator at the main resonance for the complex temporal profile of modulation function. It was shown that for the finite frequency detuning of modulation function harmonics the parametric instability is saturated with the finite amplification of oscillations excited.

Secondly, in the presence at modulation spectrum the harmonic having the exact parametric resonance the saturation of instability is absent. The instability growth rate depends significantly on this harmonic phase and for some special choice of this phase may be very small or even equal zero.

Thirdly, it is possible that the parametric instability becomes reversible when the oscillations growth will replaced by their damping.

Thus the choice of modulation function parameters determined by the function g(t), will control the parametric instability development at its linear stage, in particular, it is possible to obtain the desirable value of oscillation amplitude, the duration of amplitude amplification, to change the temporal profile of oscillations amplitude increase.

Reference

1. Galeev A.A., Sagdeev R.Z. Voprosy teorii plazmy, Moscow, Gosatomizdat,1973, No 7, p.3.
2. Shevchenko V.I., Sagdeev R.Z., Galinsky V.L., Medvedev M.V. Fizika Plazmy, 2003, v.29, No 7, p.588.
3. Moiseev N.N. Asymptotioc methods of nonlinear mechanics, Moscow, Nauka, 1969, 379 p.
4. Erokhin N.S., Mikhailovskaya L.A., Erokhin N.N. Some examples of exact solutions of mathematical models describing the continuos media oscillations. Preprint IKI RAS, Pr-

2109, Moscow, 2005, 14 p.