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CAUCHY PROBLEM FOR TECHNOLOGICAL CUMULATIVE CHARGE DESIGN

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The cumulative liner velocity independence assumption from time in its work under detonation products is made on a foundation to the flat-radial Orlenko-Stanukovitz's schema about cumulative charge operating. The Cauchy problem for ordinary differential equation of first order is formulated relative to an unknown function of the technological cumulative charge geometry. The equation solution allows constructing a technological profile of a cumulative liner at a given profile of the charge body

1. Introduction

The availability of a velocity gradient lengthwise of cumulative jet is the factor of decrease of operation performance of a shaped charge on defocused distances. If the velocity gradient can be decreased or got to zero then the charge performance of operating will be increased in a large distances from the target. This is advantage of a long-focus shaped charge.
It is possible to optimize charge operating with intent by usage of different profiles for part of the charge. The flat-radial Orlenko-Stanukovitz's schema for simulation of a cumulative action allows contacting between geometry of charge elements and their mechanical performances. The solution of a problem for optimized element of a charge is reduced to determination of a profile of a surface of this element, at the given performances and profiles of other elements. In result it also is the Cauchy problem for a differential equation. The utilization of such problem takes place at designing of new ammunition at geometrical limitations for its construction. The 'external' limitations are a cylindrical profile of the shell in artillery ammunition for example. The 'internal' limitation is technological conical liner of the cumulative charge.

2. A Cauchy problem

Let's analyze a process of the cumulative liner reduction in following reports of the writers [1,2] - fig.1. In the Cartesian *zOy* system the equations of curves are given, they determinate elements surfaces of a shaped charge at rotation around an axis *Oz*. The equation $y_1=F(z)$ is for an external surface of the shell. The equation $y_2 = \Phi(z)$ is for an

internal surface of the shell. The equation $y_3 = \varphi(z)$ is for an external surface of a cumulative liner. The equation $y_4=f(z)$ is for an internal surface of a cumulative liner. The equation $y_2 = \Phi(z)$ simultaneously describes an external surface and equation $y_3 = \varphi(z)$ describes an internal surface of an explosive charge.
The functions $y_1(z)$, $y_2(z)$, $y_3(z)$ and $y_4(z)$ are bounded they are continuous and have continuous firs derivatives and they execute following conditions:

$$F(z) \geq \Phi(z) \geq \varphi(z) \geq f(z);\ h_2 > 0;\ z \in [0; h_2];\ y \geq 0.$$

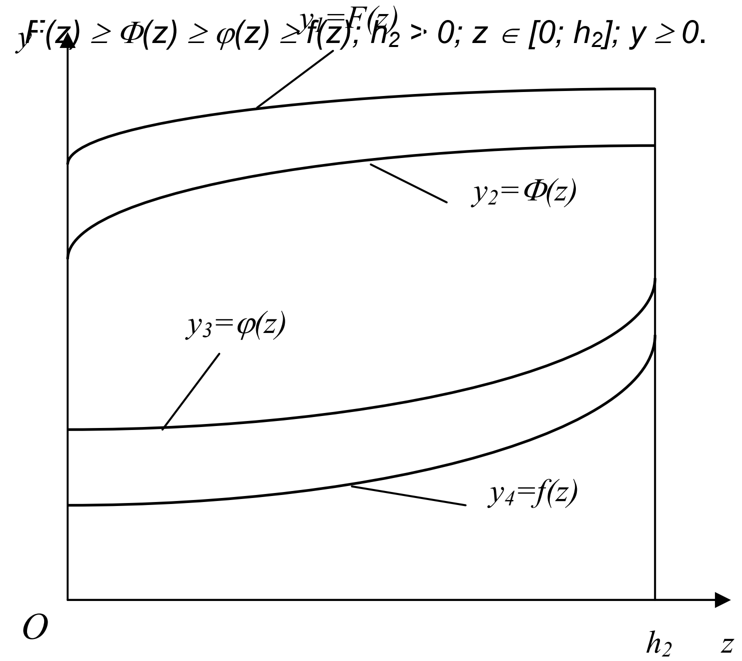

Fig.1. Cumulative charge schema and its main identifications.

It is accepted the explosive charge is isotropic and homogeneous (we leave out technological defects). The front of a detonation wave moves on mass of explosive charge from left to right and it is a flat and perpendicular to axis *Oz*. In time $t = 0$ the detonation wave reaches top of a cumulative liner and at $t > 0$ the wave are moving along a liner with detonation velocity *D*.
The schema of a cumulative liner reduction is utilized on fig.2 [3]:

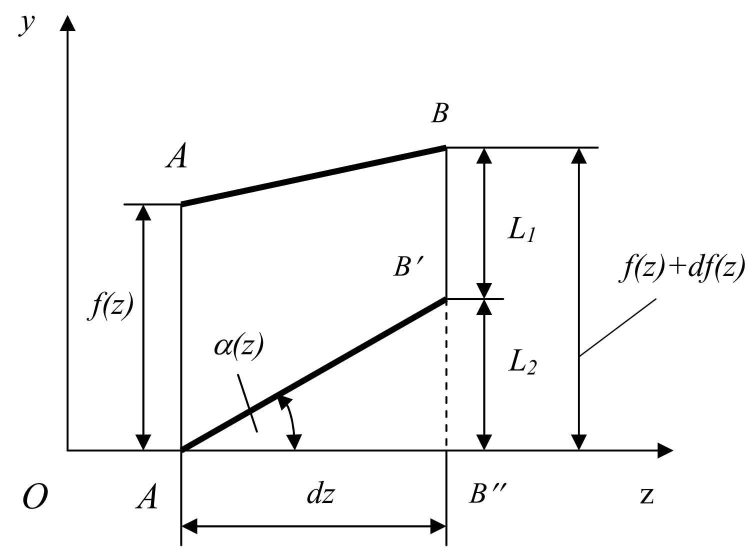

Fig.2. Reduction schema and cumulative liner collapse.

The elementary part of a liner with coordinate *z* and length *dz* is selected. In time *t = z/D* the point *A* of this part starts moving to a charge symmetry axis with velocity $W_0(z)$. During time *dt = dz/D* while the detonation wave will reach a point *B* the point *A* will be displaced in a direction of axis *Oz* on distance *dy* which one determinates itself under the formula [3]:

$$dy = W_0(z)dt = W_0(z)\frac{dz}{D} \tag{1}$$

and will be on distance from axis *Oz*:

$$R = f(z) - dy = f(z) - W_0(z)\frac{dz}{D}. \tag{2}$$

We make an assumption. The velocity $W_0(z)$ does not depend on time and it is a function only coordinate *z* [1,2]. Then point *A* will pass a distance R for time:

$$T = \frac{R}{W_0(z)} = \frac{1}{W_0(z)}\left[f(z) - W_0(z)\frac{dz}{D}\right]. \tag{3}$$

For the same time the point *B* is displaced in a direction of an axis *Oz* with velocity $W_0(z) + dW_0(z)$ and point *B* is traveled a path:

$$L_1 = T\left[W_0(z) + dW_0(z)\right] = \left[f(z) - W_0(z)\frac{dz}{D}\right]\frac{W_0(z) + dW_0(z)}{W_0(z)} \tag{4}$$

and *B* will be from an axis *Oz* at distance L_2:

$$L_2 = \left[f(z) + df(z)\right] - L_1. \tag{5}$$

The collapse angle of a cumulative liner part on fig.2 is determined under the formula:

$$tg\alpha(z) = \frac{L_2}{dz} = \frac{f(z) + df(z) - \frac{1}{W_0(z)}\left[f(z) - W_0(z)\frac{dz}{D}\right]\left[W_0(z) - dW_0(z)\right]}{dz} \tag{6}$$

or

$$tg\alpha(z) = \frac{df(z)}{dz} - \frac{f(z)}{W_0(z)}\frac{dW_0(z)}{dz} + \frac{W_0(z)}{D}.$$

The reduced velocity $W_0(z)$ of an elementary part of liner with length *dz* is connected to cumulative jet velocity $W_1(z)$ in considered cross-section *z* by a kinematics proportion [1,2]:

$$W_0(z) = W_1(z)tg\frac{\alpha(z)}{2} = \frac{k_i D}{2}\sqrt{\beta(z)\left[2 + \beta(z)\right]^{-1}}. \tag{7}$$

Then

$$tg\alpha(z)=\frac{df(z)}{dz}-\frac{f(z)}{W_1(z)tg\frac{\alpha(z)}{2}}\left[\frac{dW_1(z)}{dz}tg\frac{\alpha(z)}{2}+W_1(z)\frac{d(tg\frac{\alpha(z)}{2})}{dz}\right]+\frac{W_1(z)tg\frac{\alpha(z)}{2}}{D}. \quad (8)$$

If we shall demand the condition $W_1(z) = W_1 = const$, we take following equation:

$$tg\alpha(z)=\frac{df(z)}{dz}-f(z)\frac{1}{\sin\alpha(z)}\frac{d\alpha(z)}{dz}+\frac{W_1}{D}tg\frac{\alpha(z)}{2}. \quad (9)$$

In an equation (9) functions *tg (z), sin (z),* $tg\frac{\alpha(z)}{2}$ и $\frac{d\alpha(z)}{dz}$ are undefined. We show these functions so:

$$tg\alpha(z)=tg\left\{2arctg\frac{D}{2W_1}\sqrt{\beta(z)[2+\beta(z)]^{-1}}\right\} = tg[2arctg \in(z)]=\frac{2.\in(z)}{1-\in^2(z)}; \quad (10)$$

$$\sin\alpha(z)=\sin\left\{2arctg\frac{D}{2W_1}\sqrt{\beta(z)[2+\beta(z)]^{-1}}\right\}=\sin[2arctg \in(z)]=\frac{2\in(z)}{1+\in^2(z)}; \quad (11)$$

$$tg\frac{\alpha(z)}{2}=tg\left\{arctg\frac{D}{2W_1}\sqrt{\beta(z)[2+\beta(z)]^{-1}}\right\} = tg[2arctg \in(z)]=\in(z); \quad (12)$$

$$\frac{d\alpha(z)}{dz}=\frac{d\left\{2arctg\frac{D}{2W_1}\sqrt{\beta(z)[2+\beta(z)]^{-1}}\right\}}{dz}=\frac{d[2arctg \in(z)]}{dz}=2\frac{1}{1+\in^2(z)}\frac{d\in(z)}{dz}, \quad (13)$$

where $\in(z)$ is some intermediate function

$$\in(z)=\frac{D}{2W_1}\sqrt{\beta(z)[2+\beta(z)]^{-1}} . \quad (14)$$

The first derivative from $\in(z)$ is

$$\frac{d\in(z)}{dz}=\frac{D}{2W_1}\left\{\sqrt{\beta(z)[2+\beta(z)]^3}\right\}^{-1}\frac{d\beta(z)}{dz}. \quad (15)$$

Then the equation (9) accepts a following view:

$$\frac{2\in(z)}{1-\in^2(z)}=f'(z)-\frac{f(z)D}{2\in(z)W_1}\left\{\sqrt{\beta(z)[2+\gamma(z)]^3}\right\}^{-1}\beta'(z)+\frac{W_1\in(z)}{D}. \quad (16)$$

Densities of materials of the shell and the explosive charge and the cumulative liner ρ_K,,ρ_{BB},,ρ_O are constant values. Then the elementary masses of the shell, the explosive charge and the liner $M_H(z)$, $m(z)$, $M(z)$ participate in following expression so:

$$d\beta(z)=\frac{\partial\beta(z)}{\partial M_H(z)}dM_H(z)+\frac{\partial\beta(z)}{\partial M(z)}dM(z)+\frac{\partial\beta(z)}{\partial m(z)}dm(z)=$$

$$= \frac{\partial \beta(z)}{\partial M_H(z)}\left[\frac{\partial M_H(z)}{\partial F(z)} dF(z) + \frac{\partial M_H(z)}{\partial Fi(z)} dFi(z)\right] + \frac{\partial \beta(z)}{\partial M(z)}\left[\frac{\partial M(z)}{\partial fi(z)} dfi(z) + \frac{\partial M(z)}{\partial f(z)} df(z)\right] +$$

$$+ \frac{\partial \beta(z)}{\partial m(z)}\left[\frac{\partial m(z)}{\partial Fi(z)} dFi(z) + \frac{\partial m(z)}{\partial fi(z)} dfi(z)\right], \quad (17)$$

$$\frac{\partial \beta(z)}{\partial M_H(z)} = A(z) = \frac{m(z)\left[2M(z) + m(z)\right]}{2M(z)\left[M_H(z) + M(z) + m(z)\right]^2}; \quad (18)$$

$$\frac{\partial \beta(z)}{\partial M(z)} = B(z) = \frac{m(z)}{2M^2(z)} - \frac{m(z)}{2M^2(z)} \frac{M_H(z) - M(z)}{M_H(z) + M(z) + m(z)} - \frac{m(z)}{2M(z)} \frac{2M_H(z) + M(z)}{\left[M_H(z) + M(z) + m(z)\right]^2}; \quad (19)$$

$$\frac{\partial \beta(z)}{\partial m(z)} = C(z) \frac{1}{2M(z)} + \frac{1}{2M(z)} \frac{M_H(z) - M(z)}{M_H(z) + M(z) + m(z)} - \frac{m(z)}{2M(z)} \frac{M_H(z) - M(z)}{\left[M_H(z) + M(z) + m(z)\right]^2}; \quad (20)$$

$$\frac{\partial M_H(z)}{\partial F(z)} = a(z) = 2 p_k \pi F(z); \quad (21)$$

$$\frac{\partial M(z)}{\partial \varphi(z)} = b(z) = 2 p_O \pi \varphi(z); \quad (22)$$

$$\frac{\partial m(z)}{\partial \Phi(z)} = c(z) = 2 p_{BB} \pi \Phi(z); \quad (23)$$

$$\frac{\partial M_H(z)}{\partial \Phi(z)} = d(z) = -2 p_k \pi \Phi(z); \quad (24)$$

$$\frac{\partial M(z)}{\partial f(z)} = e(z) = -2 p_O \pi f(z); \quad (25)$$

$$\frac{\partial m(z)}{\partial \varphi(z)} = g(z) = -2 p_{BB} \pi \varphi(z). \quad (26)$$

Then we can receive a following equation:

$$\frac{df(z)}{dz} - E(z)A(z)a(z)\frac{dF(z)}{dz} - E(z)\left[A(z)d(z) + C(z)c(z)\right]\frac{d\Phi(z)}{dz} +$$

$$+ E(z)\left[B(z)b(z) + C(z)g(z)\right]\frac{d\varphi(z)}{dz} + E(z)B(z)e(z)\frac{df(z)}{dz} + \frac{W_1 \in (z)}{D} - \frac{2 \in (z)}{1 - \in^2 (z)} = 0, \quad (27)$$

where $E(z) = \frac{f(z)D}{2 \in (z) W_i} \left\{\sqrt{\beta(z)\left[2 + \beta(z)\right]^3}\right\}^{-1}$. (28)

The equation (27) is a differential equation of first order for unknown one of functions *F(z), Φ(z),* *φ(z)* or *f(z)* at given other three and initial conditions for an unknown function. This equation is assumed for formulation of a Cauchy problem about determination of the geometrical performance of a cumulative charge, which one ensures no-gradient forming of a cumulative jet.

This way allows supplying no-gradient forming of a cumulative jet by a solution of a Cauchy problem. The following two cases are baseline for such optimization.

3. Examples

3.1. Cumulative Liner Profile Design for No-Gradient Cumulative Jet

Let's study one of possible version of realization of a problem (27). At designing of cargo ammunition cumulative charge, it is necessary to link of geometry of the submunition shell with a profile of the camera of the projectile. This is a 'internal' limitation.

It is introduced a new designation for an equation of thickness of a cumulative liner on a normal to an axis *Oz* by the function $\delta(z) = \varphi(z) - f(z)$ ($\delta(z) = const$ is a special case). The problem is decided at the indicated assumptions and limitations above.

By analogy (27), it is possible to show

$$-\frac{2\in(z)}{1-\in^2(z)}+\frac{df(z)}{dz}-\frac{f(z).D}{2\in(z)W_i}\left\{\sqrt{\beta(z)[2+\beta(z)]^3}\right\}^{-1} \text{ X}$$

$$\text{X} \quad \left\{A(z)a(z)\frac{dF(z)}{dz}+\left[A(z)d(z)+C(z)c(z)\right]\frac{d\Phi(z)}{dz}+\right.$$

$$\left.+\left[B(z).e(z)+C(z).g(z)\right].\frac{d\delta(z)}{dz}+\left[B(z).b(z)+C(z).g(z)\right].\frac{df(z)}{dz}\right\}++\frac{W_i\in(z)}{D}=0 \qquad (29)$$

or

$$\frac{df(z)}{dz}=\left\{\frac{2\in(z)}{1-\in^2(z)}-\frac{W_i\in(z)}{D}+E(z)\left\{A(z)a(z)\frac{dF(z)}{dz}+\left[A(z)d(z)+C(z)c(z)\right]\frac{d\Phi(z)}{dz}+\right.\right.$$

$$\left.\left.+\left[B(z)e(z)+C(z)g(z)\right]\frac{d\delta(z)}{dz}\right\}\right\}Q(z), \qquad (30)$$

where $Q(z)=\{1-E(z)[B(z)b(z)+C(z)g(z)]\}^{-1}$ (31)

and the values *b(z), e(z), g(z)* are determined similarly (21) - (26) with allowance for links $\delta(z)$.

In result, the Cauchy problem is formulated for a differential equation of first order concerning an unknown function *f(z)*. The numerical solution of a problem allows constructing a profile of a cumulative liner for no-gradient forming of a cumulative jet. For its solution it is necessary to admit the hypothesis: *f(0)* - is small on value. Then the first value of an unknown function *f (z)* can be received with the help of the formula of the Euler:

$$f(z_1)=f(z_0)+tg\alpha(z_0)\Delta z. \qquad (32)$$

Fig.3 demonstrates a solution of a problem 3.1 for no-gradient cumulative jet with vary of a liner thickness $\delta(z) = \delta = const$: δ_1 = 1 mm; δ_2 = 2 mm; δ_3 = 3 mm; δ_4 = 4 mm, for a cumulative charge by constant thickness of the cylindrical shell.

Fig.4 demonstrates a solution of a problem 3.1 for no-gradient cumulative jet with vary of velocity of a cumulative jet W_1 = const: $W_{1,1}$ = 5000 m/s; $W_{1,2}$ = 7000 m/s; $W_{1,3}$ = 9000 m/s; $W_{1,4}$ = 11000 m/s; $W_{1,5}$ = 13000 m/s for a cumulative charge by constant thickness of the cylindrical shell.

But the decided cumulative liner is not technological. In this case the new cumulative liner form could make a problem in serial manufacture. That is why we have to determinate a partial solution for this problem where the limitation for cumulative liner must be a conical form and no-gradient jet.

3.2. Conical Cumulative Liner Profile Design For No-Gradient Cumulative Jet

Let's study another version of a problem realization (27). In this case the we are going required a condition for technological and conical type of liner and a cylindrical type of the

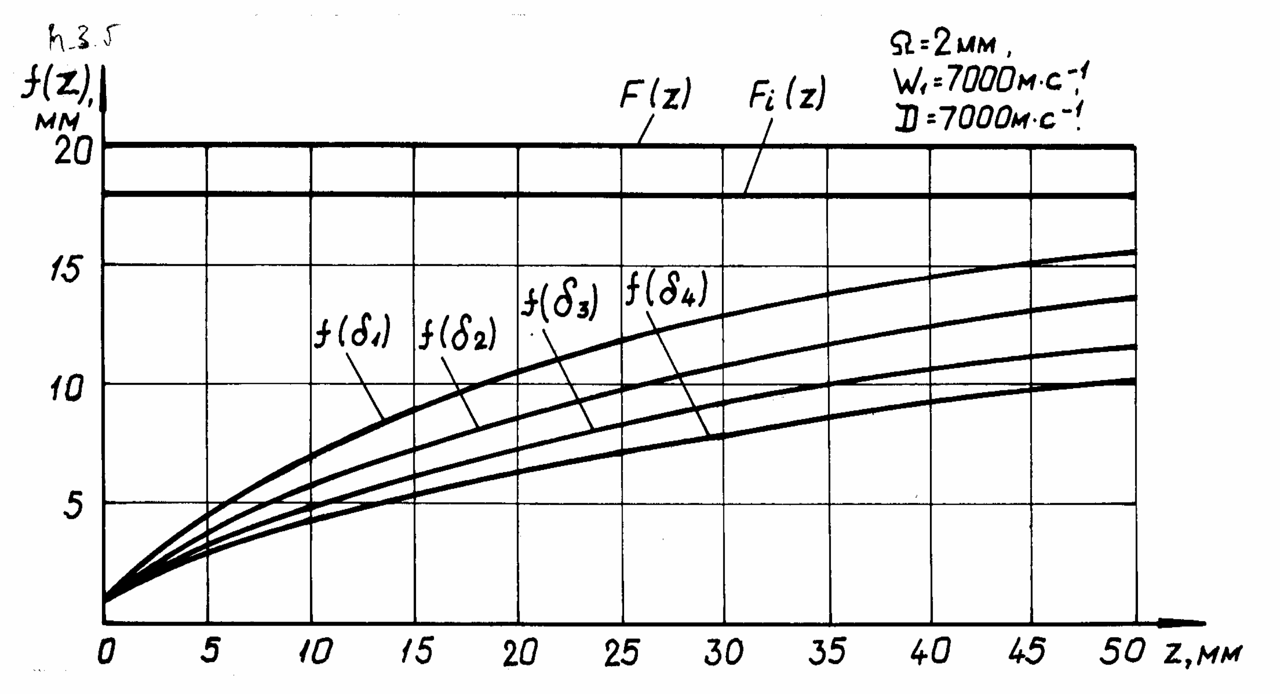

Fig.3

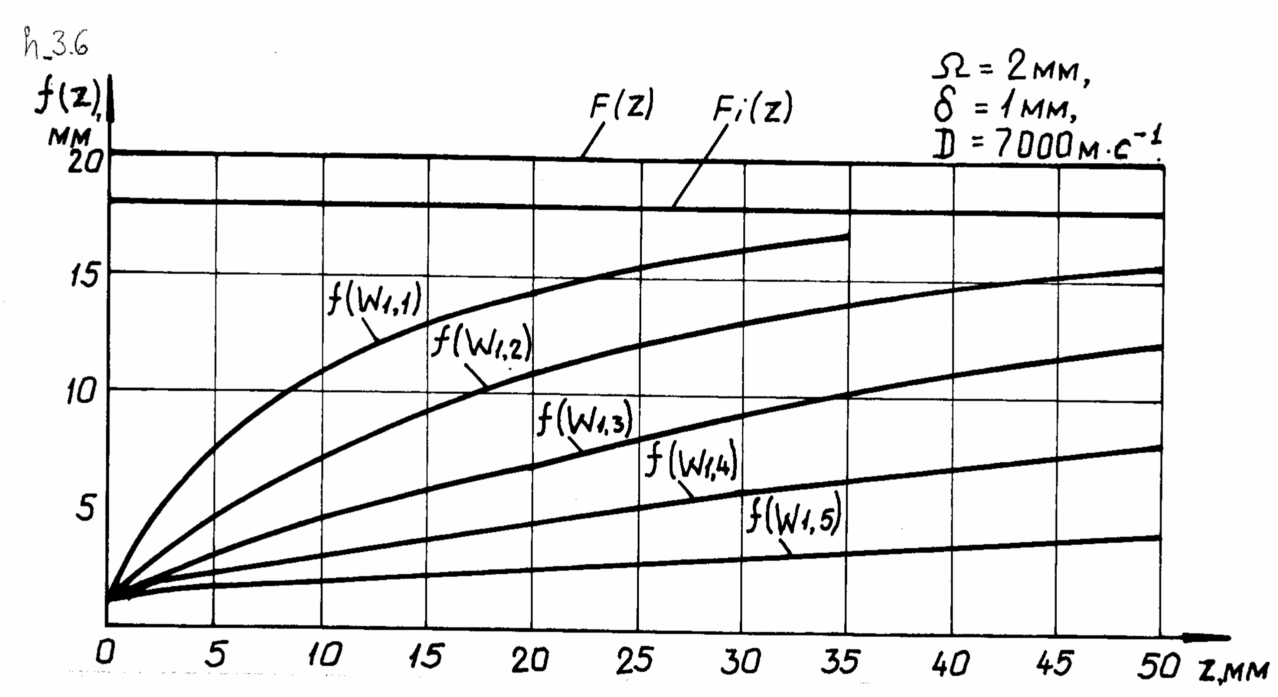

Fig.4.

geometry of the submunition shell with a profile of the camera of the projectile. This is an ‘internal’ limitation.

It is introduced a new designation for an equation of thickness of a cumulative liner on a normal to an axis *Oz* by the function *δ(z) = φ(z) -f (z.* In this case *δ(z) ≠ const* and *φ´(z) = const* and *f´ (z) = const* and *F´(x) = Φ´(x) = 0.* The problem is decided at the indicated assumptions and limitations above and by analogy (27) we can write

$$E(z)\left[B(z)b(z)+C(z)g(z)\right]\frac{d\varphi(z)}{dz}+\left[1+E(z)B(z)e(z)\right]\frac{df(z)}{dz}+\frac{W_1 \in (z)}{D}-\frac{2 \in (z)}{1-\in^2 (z)}=0 \text{ ,(33)}$$

or

$$\frac{d\delta(z)}{dx}=\left\{\frac{2.\varepsilon(z)}{1-\varepsilon^2(z)}-\frac{W_1(z).\varepsilon(z)}{D}+\frac{df}{dz}+[B(z)b(z)+C(z)g(z)]\frac{df}{dz}\right\}.Q(z) \qquad (34)$$

where $Q(z)=\left\{1-E(z)\left[B(z)b(z)+C(z)g(z)\right]\right\}^{-1}$ (35)

and the values *b(z), e(z), g(z)* are determined similarly (21) - (26) with allowance for links *δ(z).*

In result, the Cauchy problem is formulated for a differential equation of first order concerning an unknown function *δ(z)*. The numerical solution of a problem allows constructing a conical profile of a cumulative liner for no-gradient forming of a cumulative jet in case of cylindrical shell.

Fig.5 demonstrates a solution of a problem 3.2

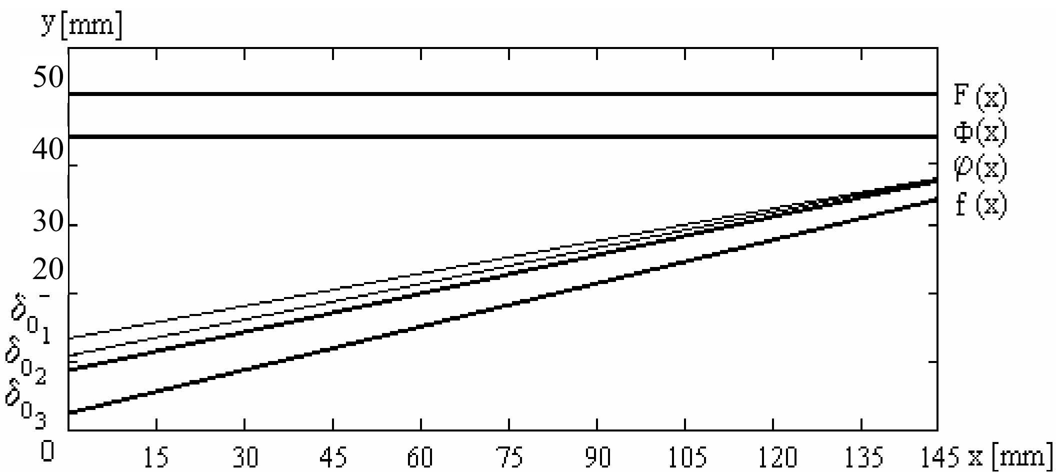

Fig. 5.

Fig. 6 shows the decision of a straight line problem for cumulative construction which we designed.

4. Conclusion

The obtained results are tested by experiment and the criterion for no-gradient cumulative jet was checked up through the depth of armor penetration in a homogeneous steel armor. At variation of distances between a cumulative charge and barrier the depth piercing was saved approximately constant, which one is the evidence of obtained theoretical results – fig 7

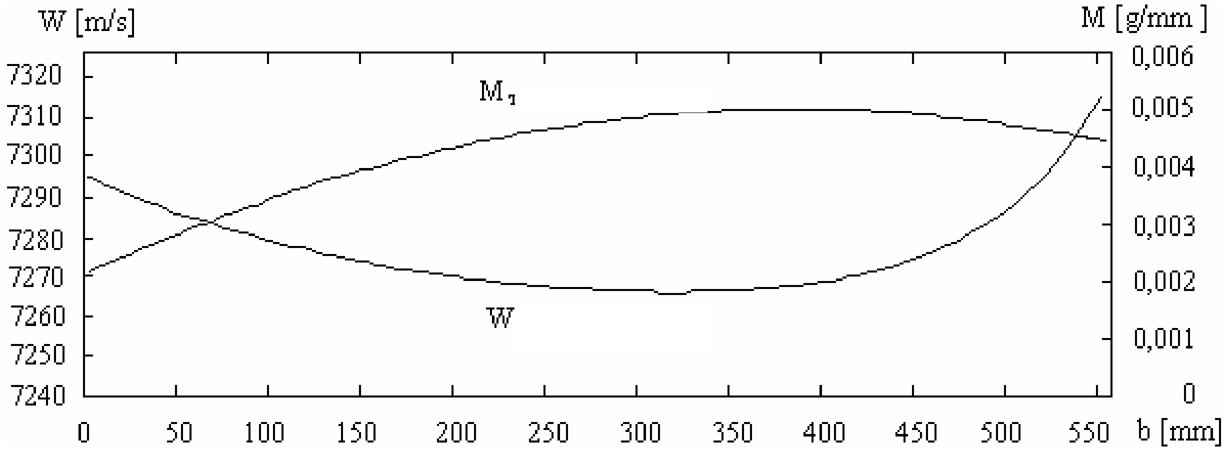

Fig. 6.

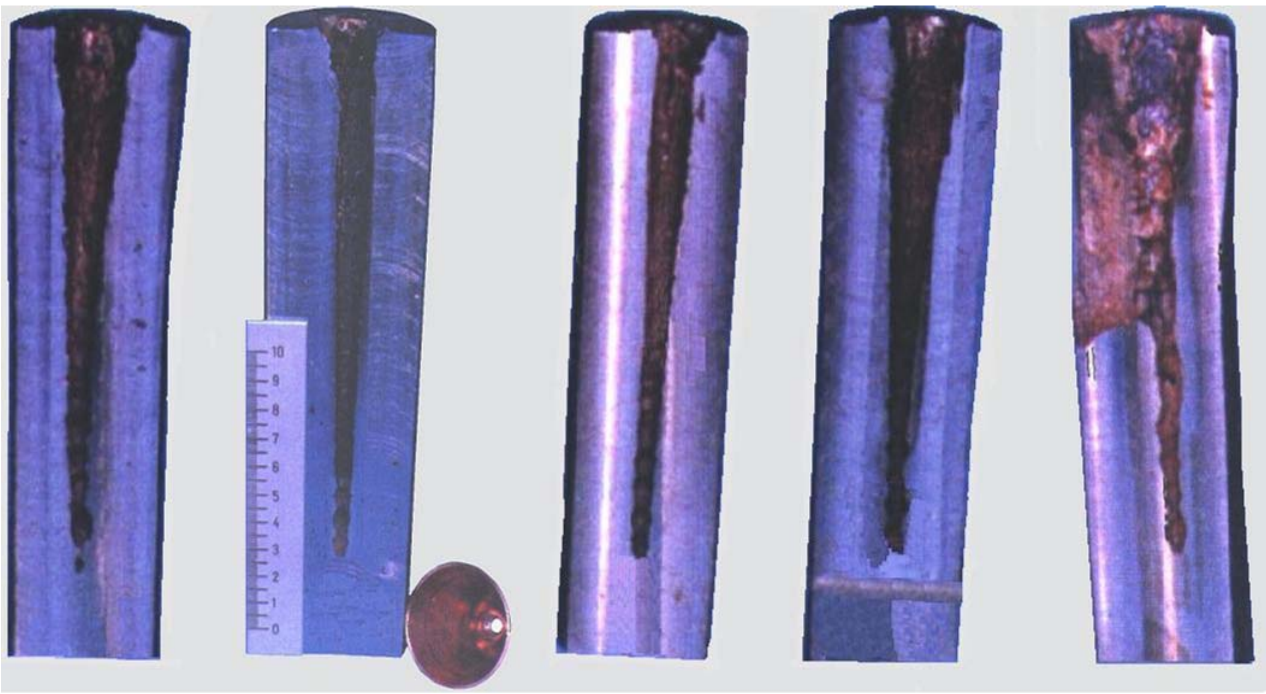

Fig. 7.

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